IMPROVEMENT OF STOCHASTIC MODELLING SKILLS FOR A SUSTAINABLE EDUCATION IN ENGINEERING

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Abstract - Deterministic and stochastic models play an important role in engineering, economics, and the natural sciences. Despite this, the development of stochastic modelling skills in engineering students is less emphasized, and this part of modelling knowledge is less well founded in secondary education. The concept of sustainable education is based on the idea that education is driven by the implication and energy of the students and teachers and their natural energy for learning is continuously renewed. In this article, we describe an approach that uses simple tools to highlight the role and importance of the stochastic approach in engineering and also serves as a model for developing application skills that equip learners with the knowledge and values needed to build a more sustainable and resilient future for all.

Keywords: stochastic modeling, sustainable education, students' abilities and skills.

1. INTRODUCTION

Sustainable education is an approach to teaching and learning that emphasizes the principles and practices of sustainability. It aims to empower learners to make informed decisions and take actions that contribute to creating a more sustainable world. The key aspects of sustainable education are interdisciplinary approach, critical thinking and problem-solving, system thinking, experimental learning, global perspective community engagement, and lifelong learning. Therefore, young generation need the ability of analysing complex problems and develop creative solutions while considering all the influence factors. For example, the success of a business plan or the return on an investment is influenced by many unforeseen circumstances such as financial conditions or the behaviour and decisions of partners and competitors. Although there is no doubt about the random nature of the outcome of the processes in the technical and natural sciences, studying the laws of nature and the connections between the natural sciences (especially physics and chemistry) lays the foundation for the deterministic way of thinking, and according to experience, this almost exclusively determines the thinking of young people.

Stochastic modeling is explicitly present in certain specific subjects, such as statistical physics, where it's included in high school curricula. However, even when it does find its place, it often remains isolated from broader educational contexts. In our experience, this leads to difficulties in introducing and using stochastic models. As a result, in undergraduate engineering courses, we suggest introducing the fundamental concepts of stochastic modeling at the outset. This approach should be based on foundational statistical tools and intuitive understanding.

One of the key elements of sustainable education is the incorporation of key environmental challenges, such as climate change, into core subjects like math, science, and art [0]. Learning the tools of mathematical analysis is essential before delving into a systematic exploration of probability theory. Yet, if we strictly insist on the superposition of theories when scheduling the topic in the curriculum, we encounter the issue that modeling, the foundation of engineering mindset, is introduced only towards the end of the educational journey, leaving insufficient time for skill development. This problem related to control theory is discussed in [1]. Our approach gives the opportunity to deal with stochastic modelling even in high school or at the beginning of the university education.

In the minds of ordinary people, including high school students, deterministic thinking is often linked to technical sciences, emphasizing precision and clear cause-and-effect relationships.

The idea that determinism exists in real systems only in the sense that inputs generally exactly determine outputs is also not sufficiently emphasized in secondary school and university education.

Indeed, it is accurate that only at a theoretical level, within a model, can one determine the output precisely through formulas when input values and system functioning are known. However, in practical studies of real processes, this ideal scenario never materializes. The presence of uncertainty stems from various factors, including noise in inputs, variability in system parameters, and unpredictable circumstances. Additionally, the description of the system itself cannot be flawless, contributing to the overall uncertainty.

An input or a parameter of a system is considered deterministic or stochastic depending on how its variability impacts the variability of the output. Capturing the variability of an output parameter (e.g., a property of a product) is a key task in quality management. Users perceive this variability and have objective or subjective expectations regarding it.

For example, if the product quality depends on a raw material property or a technological parameter in a production process, its continuous sampling and control is essential to meet the requirements. In practice, this means that we cannot neglect the variability of the given parameter, i.e. we must consider it as a random variable.

The basic concepts of reliability theory and quality assurance require the skilled application of the stochastic approach. According to interviews with university students, even top performing students often struggle to comprehend the utility and practical application of stochastic models when probability theory and statistics are discussed in isolation within mathematics courses. Typically, what is learned in mathematical statistics is not directly linked to technical systems. For sustainable engineering education, it's crucial to bridge this gap by reinforcing knowledge in engineering courses. However, this often translates into mere repetition of formulas or calculation procedures, rather than fostering skillful and creative application of probability theory and statistics. Our approach differs by integrating concepts with real-world applications from the outset, thereby ensuring that stochastic modeling becomes a vibrant and practical tool in the student's mind.

The stochastic approach does not contradict the cause-and-effect relationships assumed in technical systems. The random character doesn't lay in the relationships, but rather in the availability of information. Uncertainty arises from information gaps; when we lack complete information about a system or process, its operation and output appear random to us. Moreover, it's vital to recognize that the same phenomenon can be associated with different levels of uncertainty for different observers, contingent on their information about the phenomenon. Consequently, this can lead to different models of a problem. This concept aligns with conditional probability, where the probability of the same event varies under different conditions. In calculating conditional probability, the condition means what information we have about the outcome of the random phenomenon.

Two ideas are important in teaching stochastic models in engineering. First, the exact value of physical quantities remains a theoretical value and cannot be determined precisely through measurement. This holds true for quantities that can vary continuously within an interval; while discrete quantities, like the number of pieces, can of course be precisely determined. The fact that measurements are distorted by noise and cannot be unbiased should be a fundamental element of engineering thinking. For example, when measuring the weight of an object, the theoretical value cannot be pinpointed precisely; any mass measurement provides only an estimate.

The "accuracy" of the value provided by a particular measurement device can be determined through measurement system analysis (Gage R&R). A fundamental element of 6 Sigma process development is the examination of the adequacy of the measurement system. The 6 Sigma process development mindset and toolset provide an excellent basis for studying somatic modelling and problem solving. The tools of probability and especially mathematical statistics can be presented in a unified, practical approach.

The second crucial aspect in teaching stochastic models in engineering is clarifying the characteristics and role of the model employed for problem-solving in a given situation. In physics and engineering education, students encounter models that serve as simplifications of real-world phenomena. It's imperative for students need to see the difference between the model and reality. Usually, when discussing the learning material certain circumstances and relationships are considered "natural" without clarifying what kind of simplifications were used and what aspects were neglected to get the model. Helping students understand these distinctions enhances their ability to critically analyze and effectively apply the models in practical situations.

Students need to see that different approaches and accuracy requirements lead to different models, and the "solutions" to the problems will be different. The outcome/result is contingent upon the chosen model, and the engineer is responsible for selecting the right model in practical applications. In engineering education, we must use the principle of gradation to show models from the very simple ones to the most complex, linking the different theories presented at different levels of education.

This problem becomes particularly evident when engineering tasks are analysed using design and simulation software. These tools provide some sort of solution, but it may not always align with reality or provide acceptable solution to the engineering problem. A comprehensive and sustainable engineering education equip students with the necessary competencies to overcome the typical problem of evaluating the results of a finite element simulation. The result of the simulation can be very sensitive to the model parameters used during the procedure, so great care must be taken when using the result.

The relationship between input and output can be interpreted in a given model. Obviously, this relationship is governed by natural laws, but we can only describe it within models formulated with specific assumptions. Therefore, not only the value of the quantities, but also the description of the relationships is the result of such "estimation".

One of the overarching challenges in traditional mathematics education is its focus on calculations within mathematical models, often neglecting the broader process of problem-solving. Only a small part of this process: ***real problem*** - physical model - mathematical model - mathematical solution - physical solution - ***solution of the real problem*** is presented in the classroom. In the best case, throughout the course of education all these elements are discussed, and students develop in time all the competencies needed for problem solving.

However, most students need more than just an isolated discussion of problem-solving elements. Fortunately, in the last decades, there has been an expansion in the methodology of mathematics education to include modeling, both in academic literature and practical applications but the examples are mainly related to the deterministic approaches. Currently, Hungarian secondary schools introduce basic statistical concepts, but fail to explore models where these concepts could be naturally applied. Thus, statistical knowledge, like many other subjects, is treated as a mere item on the curriculum to be learned, without students understanding its real-world applications. and generating a sort of frustration that leads to a depletion of students' energy for learning. For a sustainable education, the energy invested into learning should turn into actual development and so, new motivational energy is generated [2].

2. LITERATURE REVIEW

The role and methods of modelling and simulation in education are widely discussed in the literature.

Study [3] proposes the combination of modelling and simulation practices along with disciplinary learning as a way to synergistically integrate and take advantage of computational thinking in engineering education. Based on a survey with the participation of 37 experts from industry and academia, it also proposes a set of modelling and simulation practices, methods, and tools. The study identifies and validates a preliminary set of modelling and simulation skills, computational thinking practices, and associated methods and computational tools needed in undergraduate engineering education.

According to [4] learning mathematical concepts and algorithms in engineering education requires solving problems in projects as well as to communicating and presenting mathematical content. Any system can be described by a mathematical model, and the models can be applied in practice because the computers allow us to solve symbolically and also numerically from different design and performance. The computational oriented mathematics education in virtual learning environments has led to new possibilities for engineering work in which mathematically complex problems solved on the computer by visualization and simulation play a central role.

Article [5] focuses on a laboratory class in which students are proposed to perform mathematical modelling of a production process, considering it as a Markov process, based on Kolmogorov equations. Students are asked to implement the theoretical solution of the model for specific numerical data and to perform computer modelling of the process of searching for the limiting probabilities of system states using the authors' virtual laboratory complex.

In the context of active innovative changes taking place in science and technology, an engineer is required to have integrative creative skills, readiness to carry out multifunctional research activities based on mathematical and computer modelling with the use of digital tools. The process of scientific and technological development based on modelling requires the improvement of mathematical foundations that allow: modelling, developing algorithms, using the computer technology apparatus, evaluating the reliability of models in quantitative estimation, analysis and optimization. All this means that the teaching of mathematical modelling, based on the integration of mathematical and applied sciences in combination with information technology, is a current trend in the development of modern engineering education.

Paper [6] reports on an interview study of mathematical modelling activities involving nine professional modelers.

The research question was How can mathematical modelling by professional mathematical modelers be characterized? The analysis of our interview data, inspired by the coding procedure of grounded theory, led us to describe three main types of modelling activities as a characterization of mathematical modelling as a professional task. In data-generated modelling, models are developed primarily from quantitative data with little or no assumed knowledge of the system being modelled, while in theory-generated modelling, models are developed based on established theory. In the third activity, model-generated modelling, the development of new models is based on established models.

The importance of the statistical sciences in modern engineering is obvious, but not necessarily its many roles. The role of technology in statistics education is also diverse and needs to be considered in the context of the student and the course. [7] considers engineering statistics education and where and how statistical technology can facilitate students' conceptual structure, statistical reasoning, and confidence.

From regression to experimental design, from SPC to MCMC and large data sets, from reliability to queuing, from risk analysis to time series and image analysis, every engineering context/field touches on some aspects of statistical thinking and techniques. Data and variability are inherent in engineering problems, both real and theoretical. Because of the diversity of statistical needs in engineering, students need an introduction to statistical thinking, concepts, and techniques that they can use immediately in real-world contexts, and a coherent and logical progression that optimizes understanding at this stage and provides a foundation for ongoing learning.

Engineering strives to "sort out messes" and "pin things down" to create products with purpose, whether tangible, systemic, or conceptual. Engineering students are not yet engineers - an aspect that unfortunately seems to be overlooked in engineering education more than in many other disciplines. [7]

The theme of the work presented in [8] is about experimentation in the classroom using modelling of real phenomena and/or simulation in higher education. The idea is to rethink mathematical modelling as a didactic strategy aimed at developing not only disciplinary but also transversal competences. A theoretical proposal of how we conceive Mathematical Modelling and methodological proposals are presented along with elements to consider these processes in the classroom.

3. COMPETENCY DEVELOPMENT WITH MONTE CARLO SIMULATION PROJECTS

In our opinion and experience, Monte Carlo simulation (MCs) is best suited for a basic introduction to the stochastic approach, offering a sustainable education to future engineers. In this article, we present the elements of our method. The ideas can be applied at different levels of complexity, from high school to bachelor's and master's courses.

3.1. Knowledge elements required for the projects

The method uses the ideas and tools of project-based learning. Another example of the project-based approach used in the Technical diagnostics course at the Faculty of Engineering of the University of Debrecen is presented in [9].

The basic tools of statistics can be presented in MCs projects. Thus, the concepts introduced are linked to concrete processes and are accepted as practical things, not only as theoretical considerations. This is the main motivation of the method.

At the high school level, the key concept is the relative frequency histogram. It is easy to understand and is based on data collected through simple observations or measurements.

Table 1 shows the concepts to be introduced at high school and university level, respectively.

Table 1. Knowledge elements required in MCs projects at high school and university level

secondary school level	university level
probability (relative frequency approach)	probability theory basics
random variable (intuitive concept)	random variable cumulative distribution function probability mass function probability density function discrete and continuous models
sample	sample, sampling methods
descriptive statistics (mean, standard deviation, median, quartiles, box plot) relative frequency histogram	descriptive statistics (mean, standard deviation, median, quartiles, box plot) relative frequency histogram
	model fitting (probability plot, goodness-of-fit testing, chi-square test) confidence intervals, hypothesis testing

3.2. Steps of the classroom projects

Step 1: *Choose a system or process that can be analysed using the MCs method*. Preferably it is a technical or business system or process but in the course project it can also be any process from the everyday life.

Step 2: *Understand the operation of the system or process under investigation (e.g. a production process) in general*. It is required to choose a system or process that the team members can really understand; a project topic is acceptable if the team members are familiar with it. Therefore, an example from their professional studies or experience is the best choice.

Step 3: *Define the (numerical) output(s) you want to study (e.g. production time needed to produce a certain number of products)*. Only a numerical output that can be correctly determined by the team members is acceptable.

Step 4: *Identify the relevant system or process elements, parameters, and data, furthermore the links to the output*. Based on the understanding of the system, it is recommended to prepare a flow chart with the relevant elements. It is useful when explaining the project to the class in the presentation.

Step 5: *Identify the inputs that determine the output and give the relationship between them*. Students need to identify the inputs they want to use in the simulation. To do this, they must consider all the system inputs that affect the output and classify them as given (deterministic) or random (stochastic). Finally, they must reduce the number of inputs to a manageable level. The course project must use at least five different types of random input data (quantities).

Step 6: *Take a sample for all random inputs*. The data source can be a measurement, observation, or any database on the Internet. In the classroom project it is also acceptable to use assumed data.

Step 7: *Generate a relative frequency histogram and/or fit a model for all random inputs, if possible*. In general, the output sample does not show any particular characteristic, in which case the relative frequency histogram is used to generate random numbers rather than a fitted model. If relevant data are not available, the distribution can be assumed to be known (chosen by the students) in the course projects.

Step 8: *Generate a large number of random inputs (>1000) and calculate the outputs*. Students must study the use of the random generation tools in any software. Any software can be used, but the presentation must explain the details of the application and demonstrate the results provided by the software.

Step 9: *Analyse the output sample statistically, mainly using the tools of descriptive statistics, e.g. the relative frequency histogram*. Based on the output sample, the students have to answer some questions posed by themselves or by the teacher about some probabilities, confidence intervals or statistical tests (e.g. state the probability that the production time needed to produce a given number of products does not exceed a given time).

Step 10: *Present the results to your peers and answer their questions about the details of the project*.

3.3. Educational goals and benefits of the projects

In our method, there are a variety of pedagogical objectives due to the application of MCs. Actually, it is not among the main topic of the course, but this application is approved to be the optimal way to repeat or relearn the basic concepts and essential ideas of stochastic modeling. Table 2 shows the educational objectives related to the different activities.

In Step 1 the topic of the project was intentionally left unspecified, only sample case studies were presented. It is a good way to check the understanding of the theoretical concepts to ask students to find a system / process that can be analyzed with MCs. In this step, we want students to use their creativity to find interesting topics that are suitable for a MCs.

Since a process approach is important for engineers, for example in process improvement projects, studying the operation of processes in the context of Steps 2-5 is a beneficial activity for the students.

Steps 6-7 are applications of statistical methods. In many cases, statistics is taught more theoretically, and the applications are demonstrated through examples with prepared data (sample). A useful extension of the study of statistics is to follow a process from data acquisition to model fitting. Students need to see the role and importance of these steps when using MCs. In practice, the usability of the analysis depends largely on the quality of the model fit to the random inputs and, ultimately, on the sampling method (data collection). At this point, a key topic in statistics, the sampling method and the concept of representativeness, can be discussed.

At the high school level, students can carry out a project, for example, on the sales in the school cafeteria, characterizing the customers and determining the distribution of the number of different types of customers.

Table 2. Educational goals linked to the different activities

Activity	Educational goal
Choosing a system / process to be analyzed.	Students must understand which type of systems / processes can be analyzed with MCs and distinguish description and simulation of systems.
Identifying the output and the relevant constant (fixed) and random system parameters and inputs.	Students must understand the steps and the connections among them. A systemic approach is required.
Data acquisition	Planning and conducting sampling process. Search for relevant data on the internet, use of databases.
Data presentation, creating frequency histograms	Use of the concepts of descriptive statistics.
Model fitting	Use of goodness of fit tests (e.g. probability plot or χ^2 test)
Generating random inputs	Learning software providing random number generation and analysis of a large number of cases (e.g. MS Excel)
Analysis of the sample for the output	Students must understand what kind of information and how can be gained from a sample
Presentation	Checking the understanding and the presentation skills. Encourage the communication and critical thinking in groups.

Steps 9-10 can be used to practice drawing conclusions. The nature and use of results are very different in deterministic and stochastic models. Besides understanding the idea of MCs (where and how to use them), the other key step is the statistical analysis of the output sample.

Although the questions to be answered must be defined during the planning of the simulation, new questions may arise when looking at the results of the statistical analysis of the output. The simulation makes sense if we can use it to answer meaningful business questions.

A typical problem in practice is that statistical methods are not used for their intended purpose. During the presentation of the results, it can be discussed which questions are useful to answer in the case of the examined process.

4. SAMPLE PROJECTS AND THE EDUCATIONAL EXPERIMENT

4.1. Sample projects

At the beginning of the Reliability chapter, a few case studies are presented as an introduction to the ideas and steps of simulation. Some examples are given below.

P1 – Net energy consumption of a building equipped with a solar system

A building is supplied with electrical energy from the grid and from a solar system. For simplicity, assume that all systems in the building are supplied with electrical energy (e.g., heating, cooling, air conditioning). Electricity is drawn from the grid when the solar system is unable to meet the current demand. And if the solar system's output is greater than the demand, the excess energy is fed back into the grid.

Fix the technical parameters of the solar system calculate the electricity production of the solar system using the number and distribution of sunshine hours and other random temperature and weather conditions on a daily basis for a year. Calculate the electricity demand of the building using the number of people living in the building, their living habits, and weather conditions on a daily basis for one year. Determine the net electricity demand for one year.

P2 – Bike rental system capacity

As part of a sustainability program, a bicycle rental network has been set up in a city. The network is characterized by the time needed to cover the distance between stations. In the project it is acceptable to calculate with 4 stations (S1,…,S4).

Assume that initially there are 10 bicycles available at each station.

Use MCs to analyze the operation of the bike rental system statistically.

Assume that the time taken to travel between stations is normally distributed with the mean and standard deviation given in Tables 3 and 4.

Table 3. Mean of the trip time between the stations (in minutes)

	S1	S2	S3	S4
S1	-	12	23	9
S2	-	-	18	14
S3	-	-	-	16
S4	-	-	-	-

Table 4. Standard deviation of the trip time between the stations (in minutes)

	S1	S2	S3	S4
S1	-	4.2	6.1	3.3
S2	-	-	2.6	3.7
S3	-	-	-	4.9
S4	-	-	-	-

Suppose that the time between to desired stating time follows exponential distribution with $\lambda = 6$ min.

The probability that the next desired trip is from Start Station SSi to End Station ESj is given in Table 5.

Table 5. Probability of trips as a next desired trip

	ES1	ES2	ES3	ES4
SS1	-	0.06	0.05	0.08
SS2	0.09	-	0.04	0.12
SS3	0.15	0.11	-	0.10
SS4	0.07	0.12	0.08	-

Instructions: Generate the demand for bikes by selecting a random trip with start and end station and the desired time of departure. Generate the time for each trip requested. Check if a bike is available at the start station. If not, the customer will have to wait until one is available. Simulate the process for a period of 8 hours. Calculate the number of trips made, the total time of the trips and the utilization of the network.

P3 – Travel time from home to the faculty by bike

In order to decide to buy and use a bicycle to travel from home to the faculty, a student needs to know the statistical parameters of the travel time.

The route is given with the number of traffic lights, the probability that the cyclist has to stop at the traffic lights and the waiting times as random variables.

The travel times between the traffic lights in normal conditions are given as random variables. These times are influenced by the weather conditions and the current fitness of the rider.

Suppose that there are 6 traffic lights (TL1,..,TL6) on the route. The probability that the cyclist has to stop at the traffic lights is given in Table 6.

Table 6. The probability that the cyclist has to stop at the traffic lights

traffic light	TL1	TL2	TL3	TL4	TL5	TL6
probability	0.41	0.22	0.65	0.33	0.15	0.18

The waiting times are exponentially distributed random variables with parameters given in Table 7.

Table 7. The parameters of the waiting times at the traffic lights (minutes)

traffic light	TL1	TL2	TL3	TL4	TL5	TL6
parameter λ	1.2	3.5	2.8	3.6	2.2	1.8

The travel times between the traffic lights are supposed to be normally distributed random variables with the mean and standard deviation given in Tables 8 and 9.

Table 8. Mean of the travel time between the traffic lights (minutes)

	TL1→ TL2	TL2→ TL3	TL3→ TL4	TL4→ TL5	TL5→ TL6
mean	8.6	10.3	5.1	4.4	6.7

Table 9. Standard deviation of the travel time between the traffic lights (minutes)

	TL1→ TL2	TL2→ TL3	TL3→ TL4	TL4→ TL5	TL5→ TL6
std	1.5	3.1	0.8	1.0	1.9

The weather conditions and the current fitness of the rider are taken into account with the factors given in Tables 10 and 11, the ride time values must be multiplied by these coefficients.

Weather conditions include sunshine (sunny/cloudy/rainy), temperature (high/normal/low) and wind (headwind/no wind/tailwind). As there are 27 cases, in order to save space, only the structure of the table is shown in Table 10. with the heading and one row.

Table 10. Weather categories and the related probabilities and coefficients (only demonstration)

sunshine /rain	temp.	wind	coefficient	probability
sunny	high	tailwind	1.5	0.04

Table 11. Fitness categories and the related coefficients

	fit	normal	exhausted	feeling sick
coefficient	1.3	1.0	0.8	0.6

Use MCs to analyze the time taken to reach the faculty statistically.

Instructions: Generate the random values of the logical variables “Stop at TLi”, $i = 1, \ldots, 6$ and the waiting time values at the traffic lights, furthermore the riding times between the traffic lights.

Generate random weather and fitness conditions and multiply the riding time values with the relevant coefficients.

Estimate the probability that the travel time is less than 35 minutes.

P4 – Operation time of a production line

In a manufacturing process, the length of the operating and repair time periods is modeled with an exponential distribution with a mean of 48 minutes and 13 minutes, respectively.

Use MCs to analyze the total operating time during an 8-hour shift statistically. Estimate the probability that it is not less than 7 hours.

Instructions: Generate a sequence of random operating and repair time values for an 8-hour period and sum the operating time values. Repeat this 1000 times to obtain a sample of the total operating time per shift.

P5 – Batch manufacturing time in a one-piece flow

10 products need to be produced on a production line. There are 6 workstations on the line. The time needed to complete the specific manipulation follows a normal distribution with the mean and standard deviation given in the Table 12.

Table 12. The mean and the standard deviation of the time frame required for the operations (minutes)

	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
μ	4.8	2.2	3.6	5.0	7.6	4.4
σ	0.8	0.2	0.5	0.4	1.0	0.7

Constrain: Step $i + 1$ can start for Workpiece j if

- Step i has been completed for Workpiece j,
- Step $i + 1$ has been completed for Workpiece $j - 1$.

In one-piece flow processes, it is obvious that pieces occasionally must wait for the next step.

Use MCs to analyze the total time required to complete the 6 products statistically. Estimate the probability that the total time will not exceed 2 hours.

Instructions: Generate a sequence of random time values for all steps and parts. Use the constraint to determine the start time of the steps for all parts. (From these values, the wait time values are obtained by simple subtraction.) Calculate the total time required to complete all steps for all workpieces. Repeat this 1000 times to get a sample of the total time.

P6 – Supermarket inventory

Inventory of 3 goods G1, G2, and G3 in a supermarket is analyzed, the initial stock of these goods (at the beginning of the week) are 3000, 2500 and 200, respectively.

There are 4 types of customers (T1,…,T4) that are characterized by the likelihood that they will purchase the different types of goods when visiting the supermarket, according to Table 13.

Table 13. Characterization of the customers

	G1	G2	G3
T1	0	0.6	0.8
T2	0.5	0.5	0.3
T3	0.8	0	0.3
T4	0.6	1	0.6

Assuming that the number of customers follows a normal distribution, the mean and standard deviation are shown separately for different types of customers and different days of the week in Table 14 and Table 15, respectively.

Table 14. Mean of the number of customers

μ	T1	T2	T3	T4
Mo	250	240	150	140
Tu	280	220	200	170
We	320	310	230	230
Th	550	520	350	200
Fr	400	380	340	260
Sa	230	120	270	100
Su	200	120	190	100

Table 15. Standard deviation of the number of customers

σ	T1	T2	T3	T4
Mo	40	30	20	20
Tu	40	30	20	30
We	30	30	30	30
Th	30	40	30	20
Fr	30	20	30	10
Sa	30	20	20	10
Su	30	20	10	10

The stock is replenished by daily transports of the three types of goods. The quantity of goods transported occasionally and the probability of transporting of the different goods on the different days of a week are given in Table 16. The transported quantity of goods of G1, G2, and G3 are 800, 1000 and 1500 per transport, respectively.

Table 16. The probability of the transport of the different goods on different days

	G1	G2	G3
Mo	0.5	0	0.9
Tu	0.8	1	0.5
We	0.7	0.6	0.7
Th	0.4	0.8	0.8
Fr	0	1	0.4
Sa	0.5	0.5	0
Su	0.2	0.5	0.6

Use MCs to analyze the inventory at the end of the week statistically.

Estimate the probability that the stock of Good 1 is less than 2600.

Estimate the probability that the end-of-the-week inventory of all goods is less than 80% of the initial inventory.

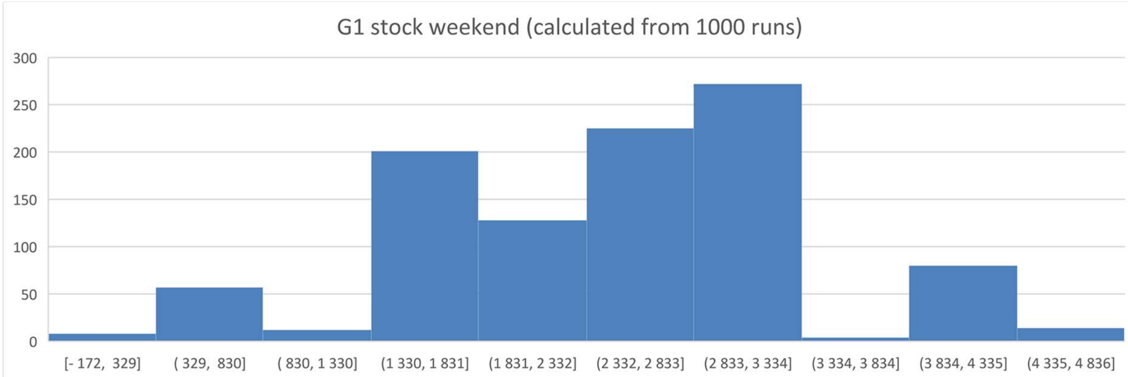

Fig. 1. A frequency histogram of the G1 stock at the end of the week

4.2. The educational experiment

The MCs projects were introduced more than five years ago; with the educational experiment detailed in this paper conducted during the fall semester of 2023. The experiment involved 60 students majoring in Engineering Management, comprising 20 Hungarian students and 40 foreign students enrolled in the faculty international program.

The initial 7 weeks of the subject "Risk and Reliability" are dedicated to risk management, while the subsequent 7 weeks focuses on reliability theory and its applications. In the practical classes of the reliability part, students engage in several projects, one of which is the MCs. This project is designed to be easily comprehensive yet complex enough to serve as a suitable project task. Students have the option to work individually or in teams of maximum 3 members.

5. RESULTS AND CONCLUSIONS

The research methodology involves analyzing numerical data derived from students' projects, primarily focusing on the time required for various steps. Additionally, informal interviews are conducted with the students to gather their experiences and insights. These findings are then synthesized and correlated with the main steps of the project, allowing for a comprehensive understanding of the students' experiences and conclusions. By linking the data analysis and interviews to the project's main steps, the research provides valuable insights into the challenges and successes encountered by students throughout the project lifecycle.

Step 1

Our observations in the experiment show that Step 1 is crucial for the success of the course projects. For some students, it was very difficult to understand the purpose of MCs and the situations in which this tool can be used effectively. Many of them could not differentiate between cases where data are available for direct statistical analysis and cases where random inputs are used to generate a sample for analysis. Several students attempted to apply probability rules, such as decision trees, to calculate some probabilities.

Similar challenges were observed across both Hungarian and foreign student groups, highlighting a common issue for many areas of engineering education: a significant disparity between students' existing knowledge (based on preliminary studies) and attitudes towards stochastic modeling, and the proficiency level required for the course.

This step requires creativity, and unfortunately, our observation revealed that the creativity of Hungarian students was generally low, and their average was much lower than that of foreign students. Specifically, when it came to identifying novel (not typical) project topics, the creativity of the foreign students (referred to as 'EN') surpassed that of the Hungarian ones ('HU') and may be characterized with the following data (estimation). Approximately

- EN-15%/HU-5% presented a very imaginative idea and do unique calculations (e.g. lap time in Formula 1 racing which depends on weather, human and technical circumstances).
- EN-55%/HU-45% worked out a common topic with significant modifications (e.g. revenue of a café or hotel per year, production or construction time, energy demand of a building)
- EN-30%-HU50% simply used an example discussed in the classroom or found on the internet and only the name of the elements and the data are changed.

Success in Step 1 is measured by the time needed to identify an acceptable project topic. The idea and goal of the MCs were repeated until all students could understand the task. In the experiment, the maximum time needed to understand the basic ideas was 4 weeks. Table 17 shows the time values in the group of Hungarian and the foreign students.

Table 17 Project topic generation time

Time (week)	Hungarian students (out of 20; %)	Foreign students (out of 40; %)
1	2 ; 10%	8 ; 20%
2	3 ; 15%	16 ; 40%
3	10; 50%	13 ; 32.5%
4	5 ; 25%	3 ; 7.5%
average time	2.9 weeks	2.275 weeks

This research has shown that student-prepared simulations should be used more frequently in engineering education to shape engineering thinking. The effect of prepared examples is much lower.

Step 2

Once students understood the basic ideas and goal of MCs and successfully completed Step 1, they encountered no significant obstacles in analysing their chosen systems. In fact, some of them opted for overly simplistic systems or processes, so they were asked to enhance the complexity of their project.

Steps 3-5

Understanding the goal of the simulation included understanding the numerical output. About a third of the students wanted to use probability or categories as output. It was time consuming to explain to these students why their approach was wrong. Some students chose systems or processes that were too complex (too many inputs or parameters) so that they could not calculate the output. Simplification was recommended in these cases. The preparation of a flow chart seemed to be useful to give an overview of the systems or processes studied.

Steps 6-7

Although the use of real data was not required, about 20% of the students collected data and used the frequency histogram or fitted model to generate the random inputs. Most of them used historical financial or weather data (e.g. stock prices or interest rates). Observation was used, for example, when the project was the analysis of "Living costs of students supported by Stipendium Hungaricum in Debrecen".

Step 8

MS Excel (random number generation and “what-if analysis”) was used by most students, some students used Matlab, MINITAB, SPSS, @risk or other software.

Step 9

The statistical analysis of the output sample was important both from a mathematical and professional point of view. About 90% used only descriptive statistics, calculated mean, standard deviation, quartiles, prepared relative frequency histogram and estimated some probabilities related to the output quantity. Three quarters of the students found the simulation a useful tool and decided to use it later in their studies (e.g. thesis).

Step 10

Presentations to the class proved to be beneficial for several reasons. Firstly, they facilitated easy evaluation of the students’ work. Secondly, they provided an opportunity for students to enhance their presentation skills. Thirdly, presentations allowed students to share their ideas regarding the challenges encountered during the project, as well as their findings and results.

As a general conclusion, this research underscores the importance of integrating student-prepared simulations, such as MCs, into engineering education to cultivate critical thinking and problem-solving skills. Furthermore, it highlights the need for ongoing support and guidance to address challenges and ensure student success throughout the project lifecycle. The collaborative sharing of experiences fostered a richer learning environment and encouraged peer-to-peer support and knowledge exchange. Overall, class presentations served as a valuable component of the learning process, promoting both academic and interpersonal skills development.

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