AC POWER LINES IMPEDANCES COMPUTATIONAL METHODS

VINȚAN M.*, MIHU P. I.*, BORLEA I.**
*University "L. Blaga" of Sibiu, E. Cioran Str. No. 4, Sibiu,
**"Politehnica" University of Timisoara, V. Parvan no. 2, Timisoara,
maria.vintan@ulbsibiu.ro

Abstract - This paper reviews the most commonly used methods for calculation of frequency-dependent impedance of overhead transmission lines: Carson's method, Sunde's expressions and the complex depth of earth return method.

Keywords: Carson's series, Sunde's expressions, complex depth of earth return method.

1. INTRODUCTION

In 1926, J.R. Carson [1] proposed a method of calculating the AC transmission line frequency-dependent impedance considering earth return. Carson's method expresses the impedance by means of an improper integral that has to be expanded into a infinite series for computation. These integrals can be either approximated using infinite series, or are evaluated using proper numerical integration methods. Due to their complex form, many researchers tried to derive simpler approximations. To represent current return through homogeneous ground, the ground can be replaced by an ideal plane placed below the ground surface at a distance equal to the complex penetration depth for plane waves. Such an approach has been proposed in [3, 5] and produces results that match those obtained from Carson's correction terms.

This paper reviews the most commonly used methods for calculation of frequency-dependent impedance of overhead transmission lines: Carson's method, Sunde's expressions and the complex depth of earth return method. There will be also presented some numerical results related to the influence of the ground soil resistivity, respectively the ground wire's influence on the zero sequence impedance of the transmission line.

2. CARSON'S METHOD

The equation for the self impedance of an overhead wire with ground return, respectively the equation for mutual impedance between two wires with ground return, proposed by Carson, are the following (a and b are conductors name) [1, 2]:

$$\underline{Z}_{aa-p} = (r_c + R_{aa-p}) + j(x_i + X_{aa-p}) \qquad (1)$$

$$\underline{Z}_{ab-p} = R_{ab-p} + jX_{ab-p} \qquad (2)$$

where r_c and x_i represents the resistance and the internal reactance of the conductor a and R_{aa-p}, R_{ab-p}, X_{aa-p} and X_{ab-p}, in $[\Omega/km]$, are given by the next expressions:

$$R_{aa-p} = \pi^2 f 10^{-4} + \Delta R_{aa-p} \qquad (3)$$

$$R_{ab-p} = \pi^2 f 10^{-4} + \Delta R_{ab-p} \qquad (4)$$

$$X_{aa-p} = \omega 10^{-3}(0.4602 \cdot \lg\frac{1}{d}\sqrt{\frac{\rho}{f}} + 1.436) + \Delta X_{aa-p} \qquad (5)$$

$$X_{ab-p} = \omega 10^{-3}(0.4602\lg\frac{1}{d_{ab}}\sqrt{\frac{\rho}{f}} + 1.295) + \Delta X_{ab-p} \qquad (6)$$

where: $\omega = 2\pi f$, f being the frequency in $[Hz]$; d represents the diameter of the conductor in $[cm]$; ρ represents the soil resistivity in $[\Omega m]$; d_{ab} represents the distance between the two conductors in $[m]$ (fig. 1). ΔR_{aa-p}, ΔR_{ab-p}, ΔX_{aa-p}, ΔX_{ab-p} in $[\Omega/km]$, are given by the following expresions [2]:

$$\Delta R_{aa-p} = \omega 10^{-4}[\frac{-5,3}{10^3}h_a\sqrt{\frac{f}{\rho}} + \frac{18,1}{10^6}h_a^2\frac{f}{\rho}(2,8 + \lg\frac{1}{h_a}\sqrt{\frac{\rho}{f}}) + \frac{111}{10^{10}}h_a^3\frac{f}{\rho}\sqrt{\frac{f}{\rho}} - \frac{81,2}{10^{13}}h_a^4\frac{f^2}{\rho^2}] \qquad (7)$$

$$\Delta R_{ab-p} = \omega 10^{-4}[-\frac{2,64}{10^3}\sqrt{\frac{f}{\rho}}D_{ab}\cos\theta + \frac{45,2}{10^7}\frac{f}{\rho}D_{ab}^2 \cdot \cos 2\theta(3,15 + \lg\frac{1}{D_{ab}}\sqrt{\frac{\rho}{f}}) + \frac{19,6}{10^7}\frac{f}{\rho}D_{ab}^2\theta\sin 2\theta + \frac{13,89}{10^{11}}\frac{f}{\rho}\sqrt{\frac{f}{\rho}}D_{ab}^3\cos 3\theta - \frac{508}{10^{15}}\frac{f^2}{\rho^2}D_{ab}^4\cos 4\theta] \qquad (8)$$

$$\Delta X_{aa-p} = \omega 10^{-4}[\frac{5,3h_a}{10^3}\sqrt{\frac{f}{\rho}} - \frac{61,7h_a^2}{10^7}\frac{f}{\rho} + \frac{111h_a^3}{10^{10}}\frac{f}{\rho}\sqrt{\frac{f}{\rho}} - \frac{238,1h_a^4}{10^{13}}\frac{f^2}{\rho^2}(3,1+\lg\frac{1}{h_a}\sqrt{\frac{\rho}{f}})] \quad (9)$$

$$\Delta X_{ab-p} = \omega 10^{-4}[\frac{2,64}{10^3}\sqrt{\frac{f}{\rho}}D_{ab}\cos\theta - \frac{15,32}{10^7}\frac{f}{\rho}D_{ab}^2 \cdot \cos 2\theta + \frac{138,9}{10^{11}}\frac{f}{\rho}\sqrt{\frac{f}{\rho}}D_{ab}^3\cos 3\theta - \frac{646,6}{10^{15}}\frac{f^2}{\rho^2}D_{ab}^4\theta \cdot \sin 4\theta - \frac{148,9}{10^{15}}\frac{f^2}{\rho^2}D_{ab}^4\cos 4\theta(3,33+\lg\frac{1}{D_{ab}}\sqrt{\frac{\rho}{f}})] \quad (10)$$

where: $\theta = \arccos\frac{h_a + h_b}{D_{ab}} = \arcsin\frac{H_{ab}}{D_{ab}}$.

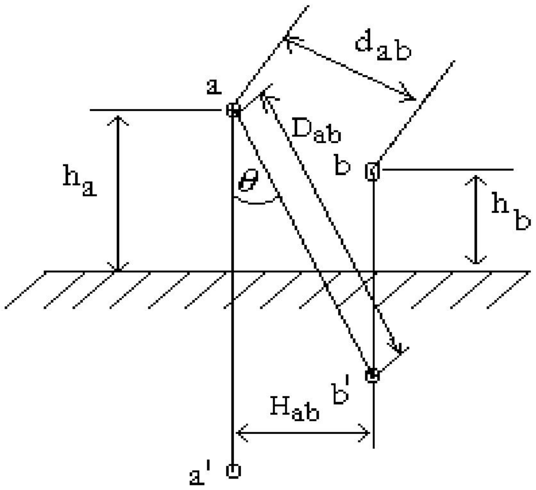

Fig. 1 Conductors *a* and *b* with there imagines; θ and distances D_{ab}, d_{ab}, H_{ab}, h_a, h_b

3. THE COMPLEX DEPTH OF EARTH RETURN

For the calculation of overhead transmission line impedances with ground return Dubanton has given explicit formulae based on the assumption that the earth return current is concentrated in a plane being in a fictitious complex depth.

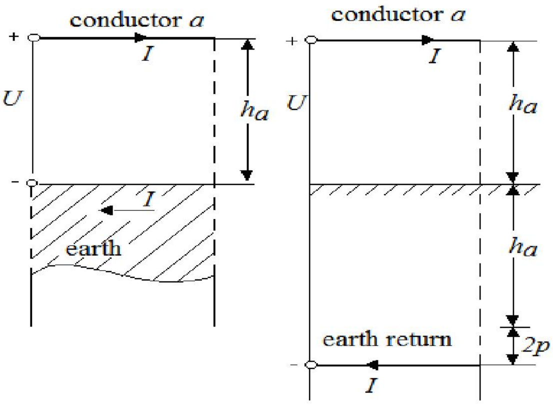

Fig. 2 The complex depth of earth return

The complex depth of earth return method, introduced by Deri [3], assumes that the current in conductor *a* returns through an imagined earth path located directly under the original conductor at a depth of ($ha + 2\underline{p}$), $\underline{p}$ being the skin depth of the ground. In other words, the earth can be replaced by a earth return conductor. It must be emphasized that this distance is a complex number since the skin depth $\underline{p}$ is a complex number. Thus, the self and the mutual impedances can be written as [3, 4]:

$$\underline{Z}_{aa-p} = (r_c + jx_i) + \frac{j\omega\mu_0}{2\pi}\ln(\frac{h+\underline{p}}{d}) \quad (11)$$

$$\underline{Z}_{ab-p} = \frac{j\omega\mu_0}{2\pi}\ln(\frac{\sqrt{(h_a+h_b+2\underline{p})^2 + H_{ab}^2}}{(h_a-h_b)^2 + H_{ab}^2}) \quad (12)$$

where: $\underline{p} = \frac{1}{\sqrt{j\omega\mu\sigma}} = \sqrt{\frac{\rho}{j\omega\mu_0}}$, in which ρ is the soil resisitivity in [$\Omega\, m$], *j* is the imaginary number, ω is the angular frequency in rad/s and μ is the ground permeability in *H/m*. Assuming that the ground permeability is equal to the permeability in free space μ_0, $\mu = \mu_0 = 4\pi \cdot 10^{-7}$ *H/m* by definition.

4. SUNDE'S EXPRESSIONS

The equation for the ground impedance of of a single-wire line with ground return, respectively the equation for mutual ground impedances between two wires with ground return, proposed by Sunde, are the following [4]:

$$\underline{Z}_{pp} = \frac{j\omega\mu_0}{\pi}\int_0^\infty \frac{e^{-2h_a x}}{\sqrt{x^2+\gamma_p^2}+x}dx \quad (13)$$

$$\underline{Z}_{pm} = \frac{j\omega\mu_0}{\pi}\int_0^\infty \frac{e^{-(h_a+h_b)x}}{\sqrt{x^2+\gamma_p^2}+x}\cos(r_{ab}x)dx \quad (14)$$

where γ_p is the propagation constant in the ground. The general expression (13) is not suitable for a numerical evaluation since it involves an integral over an infinitely long interval. Several approximations for the ground impedance of a single-wire line have been proposed in the literature. One of the simplest and most accurate was proposed by *Sunde* himself and is given by the following logarithmic function:

$$\underline{Z}_{pp}^{'} = \frac{j\omega\mu_0}{\pi}\ln(\frac{1+\gamma_p h_a}{\gamma_p h_a}) \quad (15)$$

It has been shown that the above logarithmic expression represents an excellent approximation to the general expression (13) over the frequency range of interest [4].

5. THREE-PHASE TRANSMISSION LINE, DOUBLE CIRCUIT

The zero sequence self impedance of a three-phase overhead transmission line, untransposed and asymmetrical, is given by the next expression [2]:

$$\underline{Z}_h = \frac{1}{3}(\underline{Z}_{aa-p} + \underline{Z}_{bb-p} + \underline{Z}_{cc-p}) + \\ + \frac{2}{3}(\underline{Z}_{ab-p} + \underline{Z}_{bc-p} + \underline{Z}_{ac-p}) \tag{16}$$

Considering that the three-phase overhead transmission line has one ground wire, the zero sequence impedance of the line, takind into account the presence of the ground wire, is given by the next expression:

$$\underline{Z}_{h-cp} = \underline{Z}_h - \frac{(\underline{Z}_{a-cp} + \underline{Z}_{b-cp} + \underline{Z}_{c-cp})^2}{\underline{Z}_{cp}} \tag{17}$$

$\underline{Z}_{a-cp}, \underline{Z}_{b-cp}, \underline{Z}_{c-cp}$ represents the mutual impedances between the phase conductors *a, b,* respectively *c,* and the ground wire, and $\underline{Z}_{cp}$ represents the self impedance of the ground wire with ground return.

In case that the line has two circuit, the zero sequence mutual impedance between the two circuits $\underline{Z}_{hm}$ is given by the next expression:

$$\underline{Z}_{hm} = \frac{\underline{U}_{Ah}}{\underline{I}_h} = 3\underline{Z}_{\overline{Aa-p}} + \frac{\underline{I}_{cp1}}{\underline{I}_{ah}}\underline{Z}_{\overline{A-cp1}} + \\ + \ldots + \frac{\underline{I}_{cpn}}{\underline{I}_{ah}}\underline{Z}_{\overline{A-cpn}} \tag{18}$$

In case that the line has one ground wire, expression (13) became:

$$\underline{Z}_{hm-cp1} = \underline{Z}_{hm} - 3\frac{\underline{Z}_{\overline{A-cp1}}\underline{Z}_{\overline{a-cp1}}}{\underline{Z}_{cp1}} \tag{19}$$

In case that the ground wire is in a simetricall position related to the two circuits of the transmission line, $\underline{Z}_{\overline{A-cp1}} = \underline{Z}_{\overline{a-cp1}}$ and expression (19) became:

$$\underline{Z}_{hm-cp1} = \underline{Z}_{hm} - 3\frac{\underline{Z}^2_{\overline{a-cp1}}}{\underline{Z}_{cp1}} \tag{20}$$

The zero sequence impedance of the double transmission line, as a function of the zero sequence impedance of the each circuits of the line $\underline{Z}_{hI}$, $\underline{Z}_{hII}$, is given by the next expression (fig. 3) [2]:

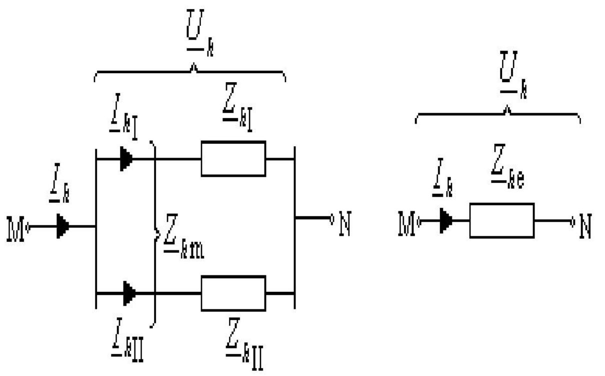

Fig. 3 Zero sequence impedance at a double transmission line

$$\underline{Z}_{he} = \frac{\underline{U}_h}{\underline{I}_h} = \frac{\underline{Z}_{hI}\underline{Z}_{hII} - \underline{Z}^2_{hm}}{\underline{Z}_{hI} + \underline{Z}_{hII} - 2\underline{Z}_{hm}} \tag{21}$$

In case that the two circuits are identical, and expression (21) became:

$$\underline{Z}_{he} = \frac{\underline{Z}_{hI} + \underline{Z}_{hm}}{2} \tag{22}$$

6. COMPARISON OF THE CALCULATION RESULTS

In order to illustrate the calculations results, it was considered a $220kV$ three-phase transmission line, double circuit, with one aluminium-steel (*AL-OL*) ground wire 160/95mm^2, respectively with one steel (*OL*) ground wire. Wires disposition is presented in figure 4.

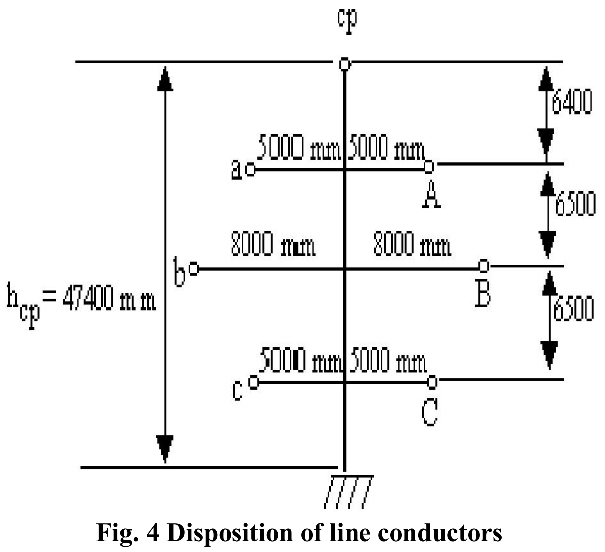

Fig. 4 Disposition of line conductors

Figures 5 and 6 presents the zero sequence resistance, respectively the zero sequence reactance of the considered line in $[\Omega / km]$, as a function of the ground resistivity in $[\Omega m]$.Values presented are considered for the case of the *AL-OL* ground wire, respectively for the case of the *OL* ground wire.

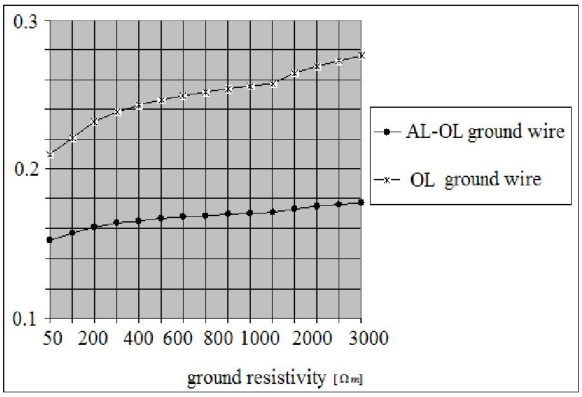

Fig. 5 Zero sequence resistance as a function of the ground resistivity

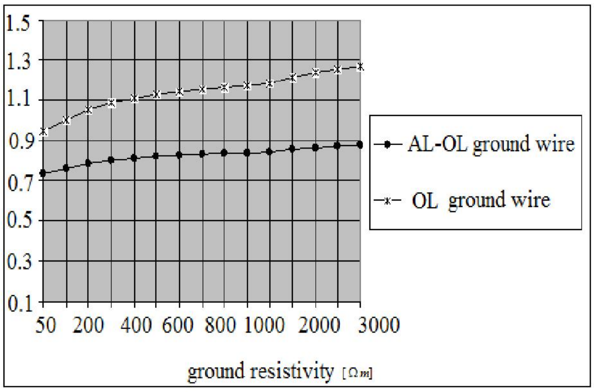

Fig. 6 Zero sequence reactance as a function of the ground resistivity

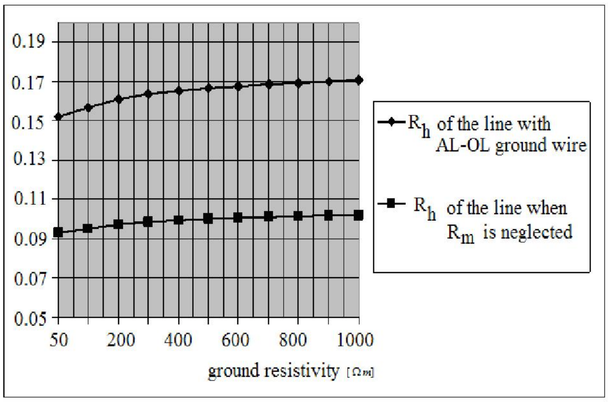

Fig. 7 Influence of the mutual resistance on the zero sequence reasistance of the considered line

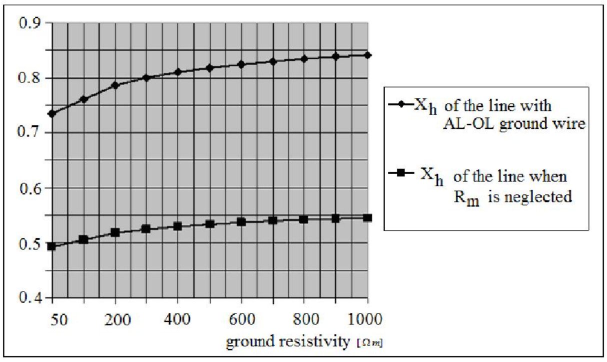

Fig. 8 Influence of the mutual reactance on the zero sequence reactance of the considered line

7. CONCLUSION

In this paper have been reviewed the most commonly used methods for calculating the frequency-dependent impedance of overhead power transmission lines [5, 6].
Carson's method is still the standard method for calculation of the frequency-dependent impedance of overhead transmission lines. It is still the only method that provides a complete analytical solution to the problem, but the solution is expressed in terms of improper integrals that have to be expanded into infinite series to allow calculation. Comparisons between numerical results obtained with Carson's expressions, respectively with complex depth of earth return method expressions for the mutual impedances, indicate that, if the distance between conductors is not big ($\beta = \frac{r_{ab}}{h_a + h_b} \leq 2$), the differences in values are less then 3%. For β<0.5 - value valid for power transmission systems, the differencies are very small, almost zero [2, 4, 5, 6]. The complex depth of earth return method is indeed a closed-form approximation of Carson's method. Besides its contribution to the approximation theory, it provides an explanation for the imagined earth return conductors. When calculating high frequency impedance using Carson's method, the term number of the infinite series must be increased in order to avoid truncation errors. On the other hand, Gary's method does not encounter this problem and is a powerful tool that can replace Carson's method [6, 7]. The distances between the imagined earth return conductors and the overhead lines are "*complex numbers*". As a result, the transmission line impedance can be written in a simple algebraic form. The main advantage of the *complex depth of earth return method,* is that it allows use of simple formulae for self and mutual impedances —formulae derived from using the images of the conductors—and, therefore, obtaining accurate results through the use of computers.
It also was studied the influence of the ground wire, respectively of the mutual coupling between the two circuits of the line on the zero sequence impedance, for different values of the ground resistivity [7].

ACKNOWLEDGMENTS

This work was supported by the Romanian National Council of Academic Research CNCSIS – UEFISCU, through the research Grant PNII – IDEI code 485/2008.

REFERENCES

[1]. Carson J. R. - Wave propagation in Overhead Wires with Ground Return, Bell System Techn. 1, vol. 5, 1926
[2]. Clarke E. –Power System Circuits Analysis, Publishing Technical House, 1979, (Translated into Romanian)
[3]. Deri A., ş.a. – The Complex Ground Return Plane. A Simplified Model for Homogeneous and Multi-Layer Earth Return, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-100, No. 8, August 1981

[4]. Rachidi F. – Transient Analysis of Multiconductor Lines Above a Lossy Ground, IEEE Transactions on Power Delivery, Vol. 14, No. 1, January 1999
[5]. Tevan G, Deri A. - Some remarks about the accurate evaluation of the Carson integral for mutual impedances of lines with earth return, Archiv fur Electrotechnik 67, by Springer-Verlag 1984
[6]. Wang Yaw-Juen, Liu Shi-Jie - A Review of Methods for Calculation of Frequency-dependent Impedance of Overhead Power Transmission Lines, Proc. Natl. Sci. Counc. ROC(A), Vol. 25, No. 6, pp. 329-338, 2001
[7]. Vințan M. - Evaluating transmission towers potentials during ground faults, Journal of Zhejiang University SCIENCE A, Zhejiang University Press, co-published with Springer-Verlag GmbH, Volume 9, Number 2/February, 2008, pp. 182-189, ISSN 1673-565X (Print); ISSN 1862-1775 (Online), China, 2008