# THE POSSIBILITY OF APPLICATION OF CHAOS THEORY IN ASSESSING THE FUNCTIONING OF ELECTRICAL DISTRIBUTING SYSTEMS

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**Abstract - The article is focused on determining the possibility to use the chaos theory in analyzing the operation of electrical distribution systems. This possibility is analyzed based on a simple model of a zone of the distribution system.**

**Key words:** chaos theory, distribution system, phase space, attractor, logistic equation

## 1. INTRODUCTION

Chaos theory has started in Henri Poincaré's tests on mathematical modeling of mechanical systems instability, in the early twentieth century. It was developed together with the improvement of computers and increase consequent of their computing power. This theory has provided the means to study complex systems. It has found applications in many diverse fields, from the most diverse, and has revolutionized scientific knowledge.

The name "chaos theory" comes from the fact that the systems that theory describes are apparently disordered, but chaos theory actually searches for internal order in these apparently random data.

The first true experimenter of chaos theory was a meteorologist Edward Lorenz. In 1961, trying to develop a mathematical model to predict the weather, he found that the evolution of the model was different in two consecutive simulations whevw the initial conditions different only in 0.000127 (Fig. 1)

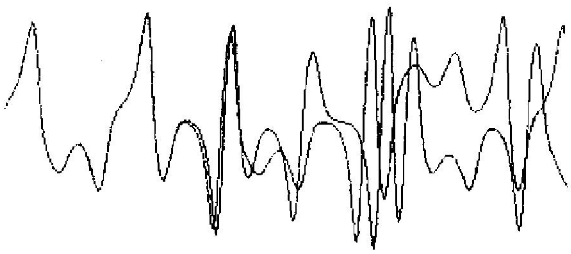

**Fig. 1. Curves representing the evolution of Edward Lorenz's model. The difference between the initial conditions is only 0.000127 [1]**

According to the dictionary, a physical system has a chaotic dynamics, if its behavior depends sensitively on its initial conditions, that is, if similar systems start up with values close to the initial conditions, they can end up in very different states.

But when science calls a system *chaotic*, it normally implies two additional requirements:

- the dynamics of the system should be relatively *simple*, in the sense that it could be expressed in the form of a mathematical expression having relatively few variables
- the geometry of the system's possible trajectories has a certain clear aspect, often characterized by a strange *attractor* [2].

## 2. CHAOTIC DYNAMIC SYSTEMS

### 2.1. Using phase space to represent physical systems dynamics in a simple form

*Phase space* is a way to view system status. If we want to visualize *the trajectory* of a body motion, we write the equations of motion (say, for a plan movement) *ox* and *oy* axes and out of both equations we will eliminate the time variable. In the phase space, there is not visualized the body's trajectory (i.e. *x*-y coordinate dependence), but the impulse dependence on the coordinate.

In physics, we operate with the representation of motion in the impulse-coordinate plan for the following reason: any mechanical movement can be traced as a succession of changes of kinetic energy and potential energy. On the other hand, the kinetic energy is determined by object's speed (impulse) and potential energy by object's position. It is easier describing the motion taking into consideration the kinetic energy (impulse) and potential energy (position) is convenient, due to the fact that total energy is conserved. Thus, in an "impulse-position" representation state, and respectively, object's energy can be better tracked. The following figure is an example in this sense.

**A simple linear oscillator, described in phase space**

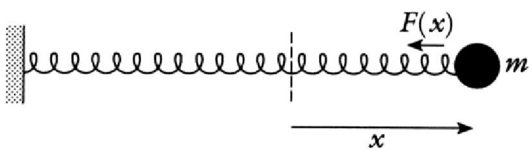


**Fig. 2. Linear harmonic oscillator**

If we recall the mathematical expressions of kinetic energy and potential energy for an oscillator, the energy conservation law is the following:

$$E_{tot} = E_{kin} + E_{pot} = \frac{p^2}{2m} + \frac{kx^2}{2} = const. \quad (1)$$

If we rewrite this relationship by highlighting the impulse and position and divide them by total energy, we get the following equation:

$$\frac{p^2}{2mE_{tot}} + \frac{kx^2}{2E_{tot}} = 1 \quad (2)$$

Or even close to what we want to obtain:

$$\frac{p^2}{2mE_{tot}} + \frac{x^2}{\left(2E_{tot}/k\right)} = 1 \quad (3)$$

This relationship that seems to be a complicated is actually the equation describing an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (4)$$

where $x$ and $y$ are the two coordinates of a point on the ellipse and $a$ and $b$ are two semiaxis of the ellipse (Fig. 3).

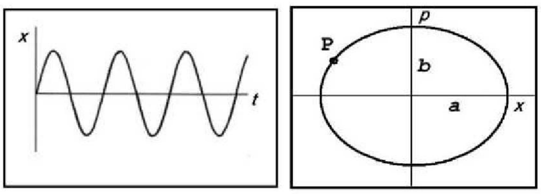


**Fig. 3. Oscillatory movement in space-time diagram and in phase space. Oscillation presented in the coordinates x - t and the path in phase space of the oscillator**

If we look at the ellipse equation and to the previously obtained equation for oscillator, we see that they both have the same mathematical form in which we can recognize two semiaxis $a$ and $b$:

$$a = \sqrt{\frac{2E_{tot}}{k}} \text{ and } b = \sqrt{2mE_{tot}} \quad (5)$$

Point P (see Fig. 3) having these coordinates, on the "impulse-position" chart is called *figurative point* and describes by its motion on the ellipse, the states the subject bears during the time of oscillation. Thus this description is done in phase space and the ellipse is called the trajectory in phase space.

Unlike trajectory in phase space for the fall of bodies, the trajectory in phase space for oscillatory movement (undepreciated) is an ellipse, ie a closed curve. An interesting property can be observed if we calculate the area of this ellipse. Ellipse area is given by Area= $\pi \cdot a \cdot b$ (it is noted that for a circle a = b and the relationship turns into the known formula of circle's area $\pi \cdot a^2$). If we introduce the expressions $a$ and $b$ in the formula of the area described by the ellipse trajectory in phase space, we get:

$$Area = 2\pi E_{tot}\sqrt{\frac{m}{k}} \quad (6)$$

If the frequency of a harmonic oscillator is given by the formula:

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (7)$$

then, the area described by the oscillator trajectory in phase space will be:

$$Area = \frac{E_{tot}}{\upsilon} \quad (8)$$

This remarkably simple relationship shows a number of things worth to be noted:

- if total energy increases or decreases, then the ellipse which describes the motion will have a greater or less area, by proportional increasing or decreasing of the semiaxis;
- if this area should remain constant, the movement frequency and total energy remain proportionate;

From the first observation we can easily understand what happens if, for example, the oscillator loses energy, ie it is a damped oscillator (similar to all real oscillators that are not maintained).

In this case, the shape of the trajectory in phase space will be only approximately elliptical, because the loss of energy (continuous) will be felt by the continuous decrease of the two semiaxis and we'll obtain a spiral trajectory (Fig. 4).

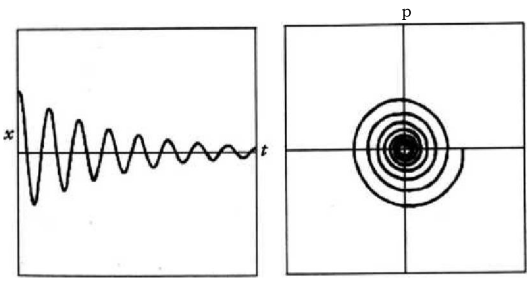


**Fig. 4 Damped oscillatory motion in space-time diagram and in phase space. Oscillation presented in the coordinates x-t and trajectory in phase space for damped oscillator.**

The description in phase space of the dynamics evolution of a system, as we have seen so far, seems to not bring something important in better understanding of the system evolution. In fact, if the system is more complicate, its description in phase space may reveal issues that would otherwise be difficult to be observed.

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## 2.2 The attractor - a feature of the geometry of the possible trajectories of chaotic systems

The attractor is characterized by a clearly defined trajectory in phase space that can be reached on equilibrium; regardless the area of the phase space the system development starts [3]. For clarity we'll consider a simple model, idealized, based on a zone of electrical distribution system.

It is assumed that in a certain area of an electrical distribution system (EDS) there are a lot of consumers (traders) who use electricity to produce products that are subsequently sold on the market in full volume. In the absence of difficulties of electricity supply, each economic agent produces and develops into a normal rhythm. We describe this situation by a dynamical number of the production lines that exist in that area of the EDS at a time certain given. We note this number - x. To write the equation that describes the evolution of the production lines number, ie the variation of x in time, we'll have to "construct an equation" that after being solver will tell us what happens to the number of technological lines. For this purpose, let's to see what are the factors influencing the increase or decrease of this number.

Because there is sufficient spare capacity of the electrical distribution lines in this area, means that the number of new production lines that will appear in a unit time (one year) will be proportional to the number of existing technological lines in the previous year. We write this as follows:

$$\frac{x_2 - x_1}{t_2 - t_1} = cx_1 \qquad (9)$$

Here $t_2 - t_1 = \tau$ is a unit of time (one year), $x_1$ is the number of technologic lines of previous year, $x_2$ is the number of technologic lines in the current year, and $c$ is a constant.

This equation is read as follow: increasing $x_2 - x_1$ the number of technologic lines per unit time $\tau$, is proportional to the number of technological lines of the previous year. The coefficient $c$ is a constant that indicates (describe) the speed of increasing number of the technological lines. If we rewrite the above equation as follows:

$$x_2 - x_1 = cx_1\tau \quad \text{or} \quad x_2 = x_1 + cx_1\tau \qquad (10)$$

the evolution can be calculated and interpreted simple.

The relationship shows that every year the number of technologic lines (TL) increases with the amount $cx_1\tau$. If $c=0$ the number remains unchanged in time. If c> 0, the number of lines increase, if c <0, the number will decrease. For example we choose $x_1 = 1000$ and $c = 0,01$. It means that after the first year we have $x_2 = 1000 + 0,01 \times 1000 \times 1 = 1000 + 10 = 1010$ TL. In the next generation (and we always note, every two consecutive years, previous year with $x_1$ and the year that appears with $x_2$): $x_2 = 1010 + 0,01 \times 1010 \times 1 = 1020,1$ TL. If we continue the calculations we obtain the following figures for the successive years the number of TL: 1030.301, 1040.60401, so on. After a long calculation we see that the number of TL begins to grow more rapidly: It is normal because the growth is accelerating when the capacity (and thus the profit) increases. Data can be placed on a graph, as in Fig. 5

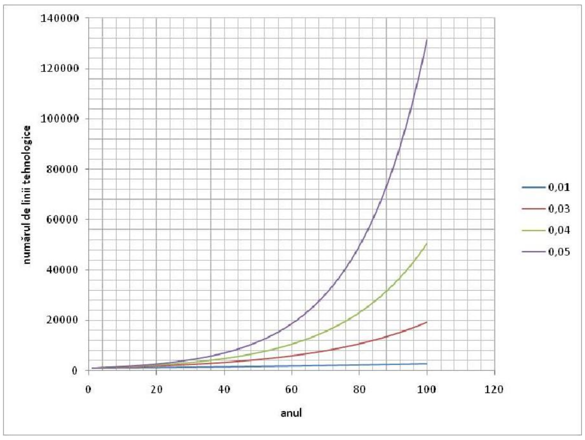



**Fig.5 Time evolution of the number of technological lines**

You can see here that for a factor of 0.01, after 100 years, the number of TL gets bigger that 2500 units, but the factor of only five times bigger, c = 0.05, after 100 years the number reached 1.5 x10 5 = 150 000 TL. Growth is very rapid at higher growth factors. What is means coefficient of 0.05? It means the emergence of five new lines in a year starting out of the first 1000. This factor is actually quite small. It would be reasonable to consider that on average appears least one TL from two initial lines after a year, ie a factor of 50% (= 0.5). In this situation is clearly that the increasing of TL number is more rapidly. It takes only 10 years to reach a total of 150 000 LT!

The studied growth mode is clearly exponentially. This model is valid for any system whose size or population growth in unit time is proportional to the sizes of the previous time unit.

But what is really happening with businesses? Businesses, such an exponential growth, will soon have a problem, will not be fully insured with electricity since transmission capacity of lines in the area will reach to limit! Businesses will suffer from a lack of energy, and growth will slow, ie the coefficient c will decrease. How does this model take into account the effect of the coefficient altering? It can be assumed that the coefficient decreases with increasing number of TL so that to a maximum number (limit) of their quantity, it will no more increase (ie c is equal to zero). If this effect is taken into account, the equation that describes the technological changes in the number of lines can be written (after some mathematical changes) in the following form:

$$X = 4\lambda X(1 - X) \qquad (11)$$

where $X$ is the number of production lines now expressed as relative to the maximum possible TL (X = 1 is the maximum value and X = 0 indicates the absence of TL) and $\lambda$ is in role of growth factor. Above equation is called logistic equation [4], describing the case of the limited amount of energy (electricity) for the economic agents in the area.

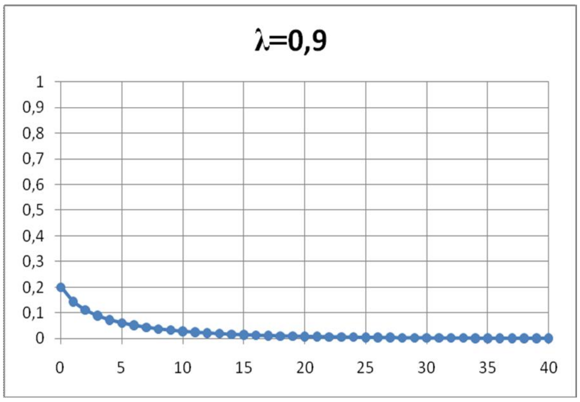
Coefficient $\lambda$ is also known as the control parameter for this model, it controls the changing dynamics of increasing technological lines. The reason of this election is not essential to describe what happens in this case. Phase space in this case is reduced to the tracking the sequence of states to each year. Obviously talking about years, we can have states which we define to one or other year and we can't talk about a fractionated "year", such as 2.3 or 15.6 years!

Therefore, the states sequence chart will draw the points that represent the states at the time, and the lines connecting the states have no other interpretation than to enable us easy to follow the developments from one state to another.

If we perform calculations on the same model as the previous one, we can get the following ongoing events of the logistic model described in phase space. The calculation can be done with a simple program written in Excel.

In graphs that are shown below is described how the number of technology lines varies depending on the time for different control parameter values.

The graphs correspond to a control parameter that has values: λ = 0.9 (represent a continuous decrease to zero)(Fig. 6,a), 2.60 (steady state with a period) (Fig. 6, b), 3, 20 (steady state with period 2) (Fig. 6, c), 3.52 (steady state with period 4)(Fig. 6, d), 4.00 (chaotic behavior, Fig. 6, e), 5.00 (stationary state to step 21 then follows a small amplification and at 34 step comes out of stability)(Fig. 6, f).

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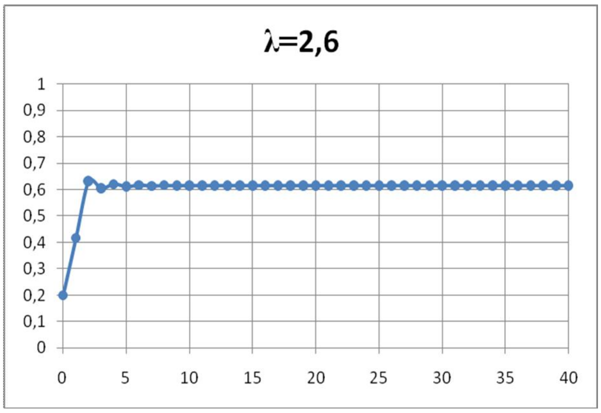


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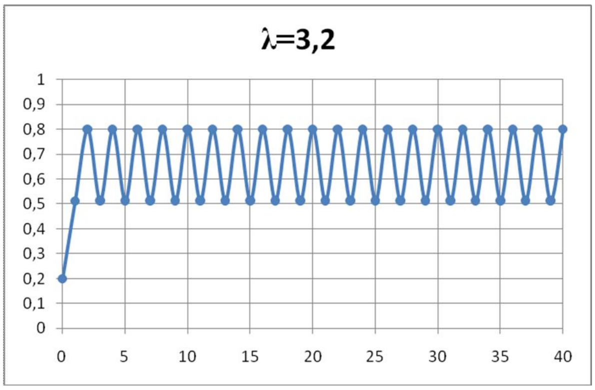
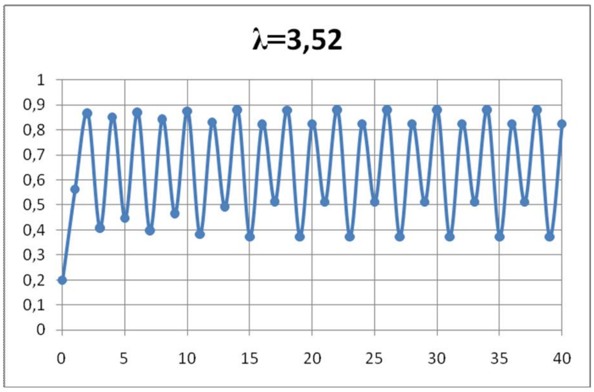
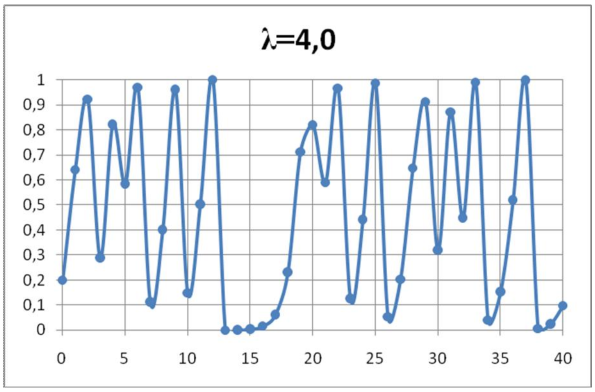



**c)**




**d)**




**e)**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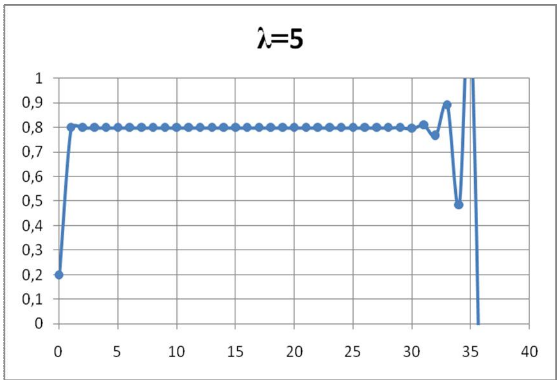


**f)**

**Fig. 6. – Evolution under the logistic model**

Generalizing, we can say that on a phenomenon that follows a logistic type model at the control parameters λ> 3.52 the system takes unpredictable developments!

In the following tables is showed how varies the evolution of the system if it begin at different initial value, $X_0=0.35$ and $X_0=0.4$. Evolution is given for three values of control parameter – λ = 2; λ = 1+√5; λ=4.

**Table 1**

| t | λ=2 | λ=1+√5 | λ=4 |
|---|---|---|---|
| 0 | 0,40000 | 0,40000 | 0,40000 |
| 1 | 0,48000 | 0,77666 | 0,96000 |
| 2 | 0,49920 | 0,56133 | 0,15360 |
| 3 | 0,50000 | 0,79684 | 0,52003 |
| 4 | 0,50000 | 0,52387 | 0,99840 |
| 5 | 0,50000 | 0,80717 | 0,00641 |
| 6 | 0,50000 | 0,50368 | 0,02547 |
| 7 | 0,50000 | 0,80897 | 0,09928 |
| 8 | 0,50000 | 0,50009 | 0,35768 |
| 9 | 0,50000 | 0,80902 | 0,91898 |
| 10 | 0,50000 | 0,50000 | 0,29782 |
| 11 | 0,50000 | 0,80902 | 0,83650 |
| 12 | 0,50000 | 0,50000 | 0,54707 |
| 13 | 0,50000 | 0,80902 | 0,99114 |
| 14 | 0,50000 | 0,50000 | 0,03514 |
| 15 | 0,50000 | 0,80902 | 0,13561 |

**Table 2**

| t | λ=2 | λ=1+√5 | λ=4 |
|---|---|---|---|
| 0 | **0,35000** | **0,35000** | 0,4000**1** |
| 1 | **0,45500** | **0,73621** | 0,9600**1** |
| 2 | 0,49**595** | **0,62847** | 0,153**57** |
| 3 | **0,49997** | **0,75561** | 0,51**995** |
| 4 | 0,50000 | **0,59758** | 0,9984**1** |
| 5 | 0,50000 | **0,77820** | 0,006**36** |
| 6 | 0,50000 | **0,55856** | 0,025**26** |
| 7 | 0,50000 | **0,79792** | 0,09**850** |
| 8 | 0,50000 | **0,52180** | 0,35**518** |
| 9 | 0,50000 | 0,80**748** | 0,91**610** |
| 10 | 0,50000 | 0,50**307** | **0,30743** |
| 11 | 0,50000 | 0,80**899** | **0,85167** |
| 12 | 0,50000 | 0,500**06** | **0,50531** |
| 13 | 0,50000 | 0,80902 | **0,99989** |
| 14 | 0,50000 | 0,50000 | **0,00045** |
| 15 | 0,50000 | 0,80902 | **0,00180** |

The first table corresponds to the evolution of the system based on the initial value $X_0=0.4$ and the second corresponds to the initial value $X_0=0.35$.

The bolded values highlight the differences in development between the two cases. What is obviously is that when the control parameter is λ=4, the evolution of the system is no predictable in the sense that small changes in initial condition completely change its further evolution.

## 3. CONCLUSIONS

Analyzing the obtained results can be extracted certain important conclusions from this simple (mathematically) experiment:

a. Although the system state is described by a simple and well defined equation (logistic equation), this system development can be predictable or unpredictable, depending on the value of control parameter, at the control parameter values greater than one critical, the system behave chaotic. We say we deal with deterministic chaos. Word "deterministic" comes from the fact that we have a correct mathematical equation describing the system, but system behavior in this regime is unpredictable.

b. System's evolution is strongly dependent (especially in the corresponding region of control parameter greater than critical) the initial condition. This feature is sometimes called “the butterfly effect”, discovered by Edward Lorenz. The effect can be expressed as follows: small changes in initial conditions due, for example, to the beating of wings of a butterfly somewhere on the globe, can lead to dramatic changes in future weather condition, in another part of the globe.

The effect is called also the sensitivity to initial conditions. If such a dramatic effect can occur even in a simple system, then to more complicated systems this effect must certainly be present.

How important are this observations can be understood from the following example. We know today that the evolution of life on a planet (for eg Earth) is subject to climatic stability on large time intervals (or planetary geological time). It was concluded that the Earth today could be the cradle of life because the climate was relatively stable over a long stretch of time. It was noted that other planets have not received such a situation. The reason why Earth has been in this favorable situation was due to the fact that at one time, cosmic time scale, the Earth captured the Moon. Capturing of Moon have stabilized Earth's rotation axis allowing a slow, but stable, evolution of climate and so the life could to appear, to stabilize, to grow and get where it is now [5]. Otherwise the movement of planets around the sun would have been so chaotic and unpredictable so that the climate in long-term was not stable, there would have drastic changes as we find that there are on Mercury, Mars or Venus, for example.

c) The cause of this sensibility to initial conditions and resulting in unpredictable behavior of the system is given by the nonlinear nature of the system. For any nonlinear physical system, we can expect that under certain conditions it to pass into a regime of deterministic chaos so its subsequent changes cannot be predicted. We insist that unpredictable system's evolution is not determined by the complexity of any system or imperfect knowledge of initial conditions, nor ignorance of the law of evolution and not by the reasons of bad mathematical calculations. Unpredictability have "organic" nature, that lies in the very nature of things or phenomena.

d. Since most real systems are nonlinear (and do not refer to linear models used to study the phenomena) aspects of deterministic chaos are ubiquitous. Only simple approximations are linear and predictable. However, some systems are unpredictable even for a short term trend (such as weather) or a long term trend (such as the cosmic phenomena). It follows that we need to know to what point we can count on a acceptable predictability and point at which we no longer have that possibility. The study of nonlinear dynamics phenomena may give an answer to this question. It required a precise knowledge of the system and taking into account those aspects which are nonlinear, although apparently these effects (or terms in the equations system) are negligible.

Is found that they are not negligible. The role of nonlinear dynamics is to indicate the limits to which approximations can give predictable results.

e. Electrical distribution systems are not an exception to those listed above. If we analyze the dynamics of the distribution system, taking into account the finest details, we find that many of the changes that occur in system, sometimes are overlapping and mutually amplifies theirs effect and approaching so the system regime at a bifurcation point where its behavior can become chaotic.

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