

# ELECTRICAL ENERGY ALIMENTATION EFFICIENCY ESTIMATION BY ANALYZING INSTANTANEUS POWER COMPONENTS CIRCULATION

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**Abstract - This paper analyzes the circulation of the instantaneous power components in non-symmetrical situations. The constant component of the active power is significant for each symmetrical components but the consumer do not receive active power on negative or zero sequence symmetrical components. Once again, the necessity of electrical circuits symmetry is demonstrated by this analyzes.**

**Keywords:** instantaneous power for positive, negative or zero symmetric components, active and reactive instantaneous power components, constant components of instantaneous power.

## 1. INSTANTANEUS POWER ON NON-SYMMETRICAL SITUATION

About non-symmetrical electrical tree phase systems an important knowledge is about instantaneous power that can give useful information about its non-symmetry. The instantaneous power in tree-phase system is:

$$p(t) = u_R \cdot i_R + u_S \cdot i_S + u_T \cdot i_T \quad (1)$$

Considering symmetrical sequence components, about an electrical tree-phase circuit supplied by symmetrical and sinusoidal electrical voltages, the instantaneous values of voltages are:

$$\begin{aligned} u_R &= U \cdot \sqrt{2} \sin \omega t \\ u_S &= U \cdot \sqrt{2} \cdot \sin\left(\omega t - \frac{2\pi}{3}\right) \\ u_T &= U \cdot \sqrt{2} \cdot \sin\left(\omega t - \frac{4\pi}{3}\right) \end{aligned} \quad (2)$$

The instantaneous values of electrical currents can be expressed dependent on symmetrical components instantaneous values:

$$\begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \begin{bmatrix} i_R^+ & i_R^- & i_R^0 \\ i_S^+ & i_S^- & i_S^0 \\ i_T^+ & i_T^- & i_T^0 \end{bmatrix} \quad (3)$$

The instantaneous power for each of the tree phase has the expression:

$$\begin{aligned} p(t) &= \begin{bmatrix} u_R & u_S & u_T \end{bmatrix} \cdot \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \begin{bmatrix} p^+ \\ p^- \\ p^0 \end{bmatrix} = \\ &= \begin{bmatrix} u_R \cdot i_R^+ + u_S \cdot i_S^+ + u_T \cdot i_T^+ \\ u_R \cdot i_R^- + u_S \cdot i_S^- + u_T \cdot i_T^- \\ u_R \cdot i_R^0 + u_S \cdot i_S^0 + u_T \cdot i_T^0 \end{bmatrix} = \begin{bmatrix} p_R^+ + p_S^+ + p_T^+ \\ p_R^- + p_S^- + p_T^- \\ p_R^0 + p_S^0 + p_T^0 \end{bmatrix} \end{aligned} \quad (4)$$

Where  $p^+$  is the positive sequence total instantaneous power, on all the tree phases,  $p^-$  is the negative sequence total instantaneous power;  $p^0$  is the zero sequence total instantaneous power.

a) Positive sequence component.

The positive sequence instantaneous power expressions, on all the tree phases, are:

$$\begin{aligned} p_R^+ &= 2 \cdot U \cdot I^+ \cdot \sin \omega t \cdot \sin(\omega t + \beta_+) \\ p_S^+ &= 2 \cdot U \cdot I^+ \cdot \sin(\omega t - 120^\circ) \cdot \sin(\omega t + \beta_+ - 120^\circ) \\ p_T^+ &= 2 \cdot U \cdot I^+ \cdot \sin(\omega t + 120^\circ) \cdot \sin(\omega t + \beta_+ + 120^\circ) \end{aligned} \quad (5)$$

Where  $I^+$  is the effective value for positive sequence component of non-symmetric current,  $\beta_+$  is the phase difference between positive sequence current component and voltage.

This instantaneous power, rel (5) can be expressed each by two components:

$$\begin{aligned}
 p_R^+ &= \frac{P^+}{3} \cdot [1 - \cos 2\alpha t] + \frac{Q^+}{3} \cdot \sin 2\alpha t = p_{RP}^+ + p_{RQ}^+ \\
 p_S^+ &= \frac{P^+}{3} \cdot [1 - \cos(2\alpha t + 120^\circ)] + \frac{Q^+}{3} \cdot \sin(2\alpha t + 120^\circ) = \\
 &= p_{SP}^+ + p_{SQ}^+ \quad (6) \\
 p_T^+ &= \frac{P^+}{3} \cdot [1 - \cos(2\alpha t - 120^\circ)] + \frac{Q^+}{3} \cdot \sin(2\alpha t - 120^\circ) = \\
 &= p_{TP}^+ + p_{TQ}^+
 \end{aligned}$$

In these expressions, the positive sequence component total active power, all three phases, is

$$P^+ = p_{Rp}^+ + p_{Sp}^+ + p_{Tp}^+ = 3U \cdot I^+ \cdot \cos \beta^+,$$

reactive power positive sequence component is

$$Q^+ = 3U \cdot I^+ \cdot \sin \beta^+.$$

$$\text{Result: } p_{RQ}^+ + p_{SQ}^+ + p_{TQ}^+ = 0.$$

b) Negative sequence component.

The expressions for negative sequence instantaneous power, three phase, are:

$$\begin{aligned}
 p_R^- &= 2 \cdot U \cdot I^- \cdot \sin \omega t \cdot \sin(\omega t + \beta^-) \\
 p_S^- &= 2 \cdot U \cdot I^- \cdot \sin(\omega t - 120^\circ) \cdot \sin(\omega t + \beta^- + 120^\circ) \quad (7) \\
 p_T^- &= 2 \cdot U \cdot I^- \cdot \sin(\omega t + 120^\circ) \cdot \sin(\omega t + \beta^- - 120^\circ)
 \end{aligned}$$

Where  $I^-$  the negative is sequence component effective value of non-symmetrical current and  $\beta^-$  is the phase difference between negative sequence current component and voltage.

This instantaneous power, rel (5) can be expressed each by two components:

$$\begin{aligned}
 p_R^- &= \frac{P^-}{3} \cdot (1 - \cos 2\alpha t) + \frac{Q^-}{3} \cdot \sin 2\alpha t = p_{RP}^- + p_{RQ}^- \\
 p_S^- &= \frac{P^-}{3} \cdot [-1/2 - \cos 2\alpha t] + \frac{Q^-}{3} \cdot \left[ +\frac{\sqrt{3}}{2} + \sin 2\alpha t \right] = \\
 &= p_{SP}^- + p_{SQ}^- \quad (8) \\
 p_T^- &= \frac{P^-}{3} \cdot [-1/2 - \cos 2\alpha t] + \frac{Q^-}{3} \cdot \left[ -\frac{\sqrt{3}}{2} + \sin 2\alpha t \right] = \\
 &= p_{TP}^- + p_{TQ}^-
 \end{aligned}$$

c) Zero sequence components.

Concerning zero sequence instantaneous power, result:

$$\begin{aligned}
 p_R^0 &= 2 \cdot U \cdot I^0 \cdot \sin(\alpha t) \cdot \sin(\alpha t + \beta^0) = \\
 &= \frac{P^0}{3} (1 - \cos 2\alpha t) + \frac{Q^0}{3} \sin 2\alpha t = p_{RP}^0 + p_{RQ}^0 \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 p_S^0 &= 2 \cdot U \cdot I^0 \cdot \sin(\alpha t - 120^\circ) \cdot \sin(\alpha t + \beta^0) = \\
 &= \frac{P^0}{3} \left( -\frac{1}{2} - \cos 2\alpha t - 120^\circ \right) + \frac{Q^0}{3} \left( -\frac{\sqrt{3}}{2} + \sin 2\alpha t - 120^\circ \right) = \\
 &= p_{SP}^0 + p_{SQ}^0 \quad (10)
 \end{aligned}$$

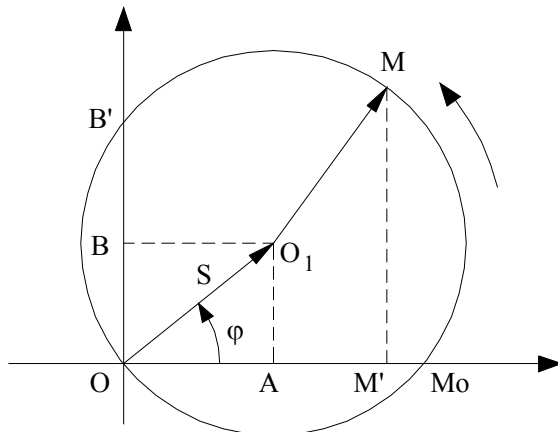
$$\begin{aligned}
 p_T^0 &= 2 \cdot U \cdot I^0 \cdot \sin(\alpha t + 120^\circ) \cdot \sin(\alpha t + \beta^0) = \\
 &= \frac{P^0}{3} \left( -\frac{1}{2} - \cos 2\alpha t + 120^\circ \right) + \frac{Q^0}{3} \left( \frac{\sqrt{3}}{2} + \sin 2\alpha t + 120^\circ \right) = \\
 &= p_{TP}^0 + p_{TQ}^0
 \end{aligned}$$

Considering an electrical circuit supplied by instantaneous voltage:  $u = U\sqrt{2} \sin \omega t$  and instantaneous current:  $i = I\sqrt{2} \sin(\omega t - \varphi)$ , the instantaneous power absorbed by this circuit is  $p(t) = u(t) \cdot i(t)$  and :

$$\begin{aligned}
 p(t) &= 2 \cdot UI \sin \omega t \cdot \sin(\omega t - \varphi) = \\
 &= UI \cos \varphi - UI \cos(2\omega t - \varphi) \quad (11)
 \end{aligned}$$

Instantaneous power can be represented [1] by a vector having a constant part and a part which turn round with the angular speed  $2\omega$ , starting at the initial time  $t=0$  from the point  $M_0$  (fig.1).

The projection  $OM'$  of the two segments:  $OO_1$  and  $O_1M$ , on the phase origin axle  $Ox$ , represents the instantaneous power  $p(t)$ . If  $\varphi$  is different from zero the circle  $O_1$  crosses complex axle  $Oy$ , in the point  $B'$  and the point  $M'$  passes in the left of origin  $O$  with every turn around of the point  $M$ . In the fig.1 the active power is represented by the segment  $OA = UI \cos \varphi = P$ , the vector  $OO_1 = UI = S$  represent the apparent power absorbed by the circuit and the segment  $OB = UI \sin \varphi = Q$  represent the reactive power.



**Fig.1. – The geometric interpretation of instantaneous power**

The projection on the phase origin axle of the vector  $O_1M$  have the value  $UI \cos(2\omega t - \varphi)$ , this value is defined

fluctuating power and determines changing sign of the instantaneous power.

For a non-symmetric three-phase circuit, the voltage and the current can be decomposed as follow (theorem Fortesque):

$$\begin{aligned} \underline{U}_1 &= \underline{U}^+ + \underline{U}^- + \underline{U}^0 \\ \underline{U}_2 &= \underline{U}^+ + a^2 \cdot \underline{U}^- + a \cdot \underline{U}^0 \\ \underline{U}_3 &= \underline{U}^+ + a \cdot \underline{U}^- + a^2 \cdot \underline{U}^0 \end{aligned} \tag{12}$$

And:

$$\begin{aligned} \underline{I}_1 &= \underline{I}^+ + \underline{I}^- + \underline{I}^0 \\ \underline{I}_2 &= \underline{I}^+ + a^2 \cdot \underline{I}^- + a \cdot \underline{I}^0 \\ \underline{I}_3 &= \underline{I}^+ + a \cdot \underline{I}^- + a^2 \cdot \underline{I}^0 \end{aligned} \tag{13}$$

The fluctuating power can [1] be calculated as follow:

$$\begin{aligned} P_f &= \underline{U}_1 \cdot \underline{I}_1 + \underline{U}_2 \cdot \underline{I}_2 + \underline{U}_3 \cdot \underline{I}_3 = \\ &= 3 \cdot \underline{U}^0 \cdot \underline{I}^0 + 3 \cdot \underline{U}^+ \cdot \underline{I}^- + 3 \cdot \underline{U}^- \cdot \underline{I}^+ \end{aligned} \tag{14}$$

A first conclusion is if in a three-phase circuit the value of fluctuating power is not zero the three-phase circuit is unbalanced. For a balanced (symmetric) circuit the fluctuating power is zero.

**2. EXPERIMENTAL MEASUREMENTS**

It is presented one of the most representative experimental measurements realized with ACE 2000 in a transformation post RAOTL, from Oradea, departure for electrical alimentation of tram redresser station. The results are for fundamental component and 3, 5, 7 harmonics, table 1.

**Table 1. The analyze of power components for PT RAOTL**

Harm onic	P	P <sup>+</sup>	P <sup>-</sup>	P <sup>0</sup>
1	53980,9	54234,9	-17,9	-271,8
3	-203,6	-1,4	-6,2	208,3
5	-320,1	-4,0	-	8,4
7	-241,9	-248,6	1,2	-7,9

In the next table there are distortion and non-symmetry parameters, for several points in electrical networks.

**Table 2. Experimental values that demonstrate presence of the non-symmetry situation**

Measurement point	Coefficient	
	K <sub>nsU</sub> [%]	K <sub>nst</sub> [%]
Centru Redresor	2,1	4,9
Centru Zamfirescu	2,2	8,2
P.T. Accesorii	0,82	6,6
P.T. Cuptoare	0,79	4,9

**3. CONCLUSION**

The power definition problem is not simple on non-symmetrical situation or/and non-sinusoidal variation for current and voltage.

For each phase, negative sequence constant component is different from zero but sum for the three phase is zero. It results that a component for one phase come back on others two phases. Oscillate components, amplitude Q<sup>-</sup> /3 of instantaneous reactive power have the same phase.

Zero sequence active power components have same circulation comparing to negative components.

Active power received by unbalanced three-phase electrical receptor is only positive sequence active power (P<sup>+</sup>), an important result for obtaining power factor in non-symmetrical situation (k<sub>p</sub>=P<sup>+</sup>/S) and appreciate the electrical energy alimentation efficiency.

The fluctuation power is an instantaneous component that has a different from zero value only on non-symmetrical situation. Its value can characterize the non-symmetry of the three-phase circuit. The graphical model of fluctuation power, fig.1, defines its influence in time, upon electrical energy alimentation.

The original contribution presented in this paper consists about instantaneous power components analyze.

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