THE DYNAMICS ANALYSIS OF A HIDRO AGGREGATE

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Abstract - This paper preents the dynamic equations describing the running of a hydropower unit and how to solve them. There are analysed the vectors, the eigenvalues and the calculation of the response to mass imbalance.

Keywords: matrices, disks, arbor elements, bearing elements

1. INTRODUCTION

The equations of motion getting is made by applying the following steps:

- Determination of kinetic energy expression Ec, of strain energy Ed and virtual work function of external forces for the system components.
- Applying Lagrange equations form:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_i}\right)-\frac{\partial E_c}{\partial q_i}-\frac{\partial E_d}{\partial q_i}=F_{qi} \qquad (1)$$

where: 1<i<N, N being the total number of system freedom degrees, q_i is the generalized coordinate of i order and, Fqi, the generalised force. The equations of motion thus obtained can be written in matrix form:

$$[\widetilde{M}]\cdot\{\ddot{\delta}\}+[\widetilde{C}]\cdot\{\dot{\delta}\}+[\widetilde{K}]\cdot\{\delta\}=\{F\}, \qquad (2)$$

In which $\left[\widetilde{M}\right]$, $\left[\widetilde{C}\right]$, $\left[\widetilde{K}\right]$, are the inertion matrices, of harmonising şi global stiffness and gyroscopic effect, of NxN system order, {δ} is the strains vector corresponding to all N freedom degrees of the system, and {F} is the vector of forces applied in the discreted mesh nodes.

2. THE EQUATIONS OF MOTION

After the quantization of the structure and choosing values of shaft speed, after calculating the static load forces in every bearing, dynamic coefficients of each bearing are determined and the global matrix of inertia $\left[\widetilde{M}\right]$ are built, of damping $\left[\widetilde{C}\right]$ and rigidity $\left[\widetilde{K}\right]$, by adding the corresponding elementary matrices. The addition of elementary matrices is done by summing up the corresponding terms of each degree of freedom separately. There result equations of motion in matrix form:

$$\left[\widetilde{M}\right]\cdot\left\{\ddot{\delta}\right\}+\left[\widetilde{C}\right]\cdot\left\{\dot{\delta}\right\}+\left[\widetilde{K}\right]\left\{\delta\right\}=\left\{F(t)\right\} \qquad (3)$$

In the above equation, the three global matrices are obtained as follows:

$$\left[\widetilde{M}\right]=\sum_{i=1}^{ND}\left[M_D\right]_i+\sum_{j=1}^{NE}\left(\left[M\right]+\left[M_A\right]\right)_j \qquad (4)$$

$$\left[\widetilde{C}\right]=\sum_{i=1}^{ND}\left[C_D\right]_i+\sum_{j=1}^{NE}\left[M_A\right]_j+\sum_{k=1}^{NL}\left[C_L\right]_k \qquad (5)$$

$$\left[\widetilde{K}\right]=\sum_{j=1}^{NE}\left(\left[K\right]+\left[K_F\right]\right)_j+\sum_{k=1}^{NL}\left[K_L\right]_k \qquad (6)$$

in which $[M_D]$,[M] and $[M_A]$ are the classical inertion matrices, $[C_D]$, $[C_A]$ and $[C_L]$ are the gyroscopic effect matrices and [K], $[K_F]$ and $[K_L]$ are the rigidity matrices,ND, NE and NL representing the number of disks of shaft elements and, respectively, of bearing. The matrices $[M_D]$, [M], $[M_A]$, [K] and $[K_F]$ are symmetrical; the matrices $[C_D]$ and $[C_A]$ are anti- symmetrical, and $[C_L]$ and $[C_K]$ are non- symmetrical. Consequently, the matrix $\left[\widetilde{M}\right]$ is symmetrical, iar the matrices $\left[\widetilde{C}\right]$ and $\left[\widetilde{K}\right]$ are non- symmetrical.

As the the quantization of the rotor in elements is done so that node j+1 of an element is the same as the node j of the next element, the three global matrices, corresponding to the bearing supported rotor, are striped matrices, with constant bandwidth and equal to eight.

If the global matrices were obtained after the quantization of the entire structure, rotor – bearings – carcass – foundation, they increase their bandwidth and their dimensions, reaching big matrices, correspondeing to thousands of nodes (fig.1) in which non-hachured spaces correspond top the spaces full of zeros

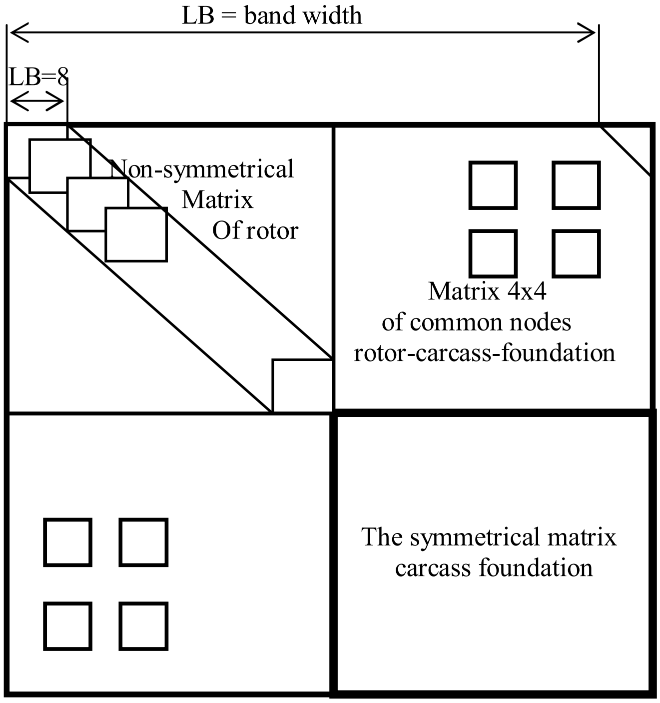

Fig. 1. Band width of global matrices

3. THE SOLVING OF MOTION EQUATIONS
3.1. Vectors and eigenvalues

The main problem in the dynamics of rotors is the determination of critical speeds, of the speed of stability loss and of the response to mass imbalance. As imbalanced masses, the disc placed obliquely on the shaft and other types of external forces, do not modify critical speeds of the rotor sistem– bearings, to calculate the free precession, the right side of equation (3) is neglected, resulting:

$$\left[\tilde{M}\right]\cdot\{\ddot{\delta}\}+\left[\tilde{C}\right]\cdot\{\dot{\delta}\}+\left[\tilde{K}\right]\cdot\{\delta\}=0 \qquad (7)$$

Using substitution:

$$\{\delta\}=\{X\}\cdot e^{\lambda t} \qquad (8)$$

where e=2,718, equation (7) reaches:

$$\begin{bmatrix} [0] & [I] \\ -\left[\tilde{M}\right]^{-1}\cdot\left[\tilde{K}\right] & -\left[\tilde{M}\right]^{-1}\cdot\left[\tilde{C}\right] \end{bmatrix}\cdot\begin{bmatrix}\{X\}\\ \lambda\{X\}\end{bmatrix}=\lambda\begin{bmatrix}\{X\}\\ \lambda\{X\}\end{bmatrix}, \qquad (9)$$

where [I] is a unit matrix.

The matrix in relation (9), is of a double order as compared with the order of the initial matrices, it is non-symmetrical. The eigenvalue problem can not be transformed into a symmetrical one, finally complex solutions will be obtained for vectors and eigenvalues. In order to reduce the size of the matrix above, processing methods can be used by choosing a base of variable vector {δ} of eigenvectors corresponding to undamped vibration, vectors corresponding to the dominant modes of vibration, generally the first eigenmodes of vibration, and their number must be at least equal to the number of the first critical speeds that are intended to be determined.

A transformation method commonly used now is the pseudo-modal method, which consists of a transformation of coordinates of the form:

$$\{\delta\}=[\Psi]\cdot\{d\} \qquad (10)$$

where the matrix [Ψ] represents the modal matrix corresponding to the first n eigenvectors, n being a lot smaller than the order of the matrices in the relation (7),

$$[\Psi]=[\{\Psi_1\},\quad \{\Psi_2\},\quad \cdots,\quad \{\Psi_n\}] \qquad (11)$$

Obtained by solving the problem of eigenvalues:

$$\left(\left[\tilde{K}^*\right]-\omega^2\left[\tilde{M}\right]\right)\cdot\{\Psi\}=\{0\} \qquad (12)$$

In the relation (12) the matrix $\left[\tilde{K}^*\right]$ is symmetrical being obtained from the matrix $\left[\tilde{K}\right]$ by annuling the non-symmetrical terms. Considering the transformation (10) and multiplying to the left with $[\Psi]^T$, relation (7) becomes:

$$[\Psi]^T\cdot\left[\tilde{M}\right]\cdot[\Psi]\cdot\{\ddot{d}\}+[\Psi]^T\cdot\left[\tilde{C}\right]\cdot[\Psi]\cdot\{\dot{d}\}+ \\ [\Psi]^T\cdot[K]\cdot[\Psi]\cdot\{d\}=\{0\} \qquad (13)$$

And after multiplications, it results:

$$\left[\tilde{M}\right]\cdot\{\ddot{d}\}+\left[\tilde{C}\right]\cdot\{\dot{d}\}+[K]\cdot\{d\}=\{0\} \qquad (14)$$

Where the new matrices obtained have the order nxn, a lot smaller than of the initial matrices. Next, to determine the first n eigenmodes of vibration of the initial system, equation (14) is brought to standard form of eigenvalues, given in relation (9), which, once solved, leads to the appearance of values and complex eigenvectors. Thus, eigenvalues will be of the form $A = \alpha \pm i\omega$, α representing the damping coefficient, and ω the eigenpulsation. For α>0 the system is unstable. Also, in the endthe eigenvectors {X} will be of a complex form, both their real and their imaginary parts, helping helping to draw ellipses precision movement along the rotor, the precision form of the shaft being generally spatial.

The main advantage of pseudo–modal method is the fact that it reduces runtime and used memory, the results being very close to those obtained by the direct method, even for a relatively small n number of retained eigenvectors, by solving the symmetric problem of eigenvalues (12).

3.2. Calculation of the response to mass imbalance

Next is presented the way in which response to imbalance can be obtained in section i, considering concentrated, excentric masses, in sections j and k. Equation (3) can also be written under the form:

$$f_1(t) = f_c \cos\Omega t + f_s \sin\Omega t \tag{15}$$

where:

$$f_c(t) = \Omega^2 \cdot \left\{ \begin{array}{c} \vdots \\ m_{nj} \cdot e_j \cdot \cos\alpha_j \\ m_{nj} \cdot e_j \cdot \sin\alpha_j \\ 0 \\ 0 \\ m_{nk} \cdot e_k \cdot \cos\alpha_k \\ m_{nk} \cdot e_k \cdot \sin\alpha_k \\ 0 \\ 0 \\ \vdots \end{array} \right\} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} node \quad j \\ \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} node \quad k \end{array} \tag{16}$$

$$f_s(t) = \Omega^2 \cdot \left\{ \begin{array}{c} \vdots \\ -m_{nj} \cdot e_j \cdot \cos\alpha_j \\ m_{nj} \cdot e_j \cdot \sin\alpha_j \\ 0 \\ 0 \\ -m_{nk} \cdot e_k \cdot \sin\alpha_k \\ m_{nk} \cdot e_k \cdot \cos\alpha_k \\ 0 \\ 0 \\ \vdots \end{array} \right\} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} node \quad j \\ \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} node \quad k \end{array} \tag{17}$$

For the calculation of mass imbalance, equation (7) has the right side different from zero and equal with $f_1(t)$:

$$[\tilde{M}] \cdot \{\ddot{\delta}\} + [\tilde{C}] \cdot \{\dot{\delta}\} + [K] \cdot \{\delta\} = \{0\} \tag{18}$$

The solution will be as disruptive force, so:

$$\{\delta\} = \{\delta_c\}\cos\Omega t + \{\delta_s\}\sin\Omega t \tag{19}$$

Replacing (19) in (18) and separating the terms in sin and cos from both members of the relation, there results the double-order system matrices compared to the initial matrices order:

$$\begin{bmatrix} [\tilde{K}] - \Omega^2[M] & \Omega[\tilde{C}] \\ -\Omega[\tilde{C}] & [\tilde{K}] - \Omega^2[\tilde{M}] \end{bmatrix} \cdot \begin{bmatrix} \delta_C \\ \delta_S \end{bmatrix} = \begin{bmatrix} f_C \\ f_S \end{bmatrix} \tag{20}$$

the vector of unknown being:

$$\begin{bmatrix} \delta_C \\ \delta_S \end{bmatrix} = \begin{bmatrix} [\tilde{K}] - \Omega^2[M] & \Omega[\tilde{C}] \\ -\Omega[\tilde{C}] & [\tilde{K}] - \Omega^2[\tilde{M}] \end{bmatrix}^{-1} \cdot \begin{bmatrix} f_C \\ f_S \end{bmatrix} \tag{21}$$

Response to imbalance in the i section is obtained from the degrees of freedom of node i, which corresponds to elements 4i-3, 4i-2, 4i-1, 4i, for the terms of cos function and n+4i-3, n+4i-2, n+4i-1, n+4i, and, for those of sin function, n being the number of degrees of freedom of the system ui, from the vector of unknown. Retaining only the two-way movement of node i we can write:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \delta_{4k-3}\cos\Omega t & \delta_{n+4k-3}\sin\Omega t \\ \delta_{4k-2}\cos\Omega t & \delta_{n+4k-2}\sin\Omega t \end{bmatrix} \tag{22}$$

The indices in relation (22) correspond to the place of the elements in the unknown vector, numbered from top to bottom. Relation (22) can also be written under the form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_c\cos\Omega t & x_s\sin\Omega t \\ y_c\cos\Omega t & y_s\sin\Omega t \end{bmatrix} \tag{23}$$

or:

$$\begin{cases} x(t) = \sqrt{x_c^2 + x_s^2}\,\sin(\Omega t + \varphi_x) \\ y(t) = \sqrt{y_c^2 + y_s^2}\,\sin(\Omega t + \varphi_y) \end{cases} \tag{24}$$

where:

$$\begin{cases} \varphi_x = arctg\dfrac{x_c}{x_s} \\ \varphi_y = arctg\dfrac{y_c}{y_s} \end{cases} \tag{25}$$

Orbit of the precession motion of the center of the shaft in section i, defined by the vector r (t), is generally in the form of an ellipse (fig2).

$$\bar{r}(t) = \bar{x}(t) + \bar{y}(t) \tag{26}$$

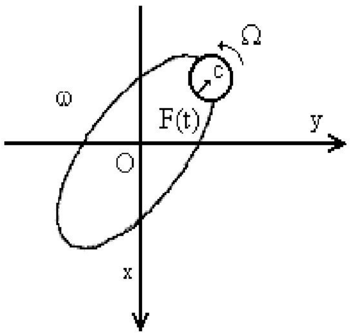

Fig. 2. Orbit of the precession motion of the center of the shaft

Noting with A and B the big semi-axis, respectively the small semi-axis of the ellipse, there result their values:

$$\begin{cases} A = r_+ + r_- \\ B = r_+ - r_- \end{cases}, \tag{27}$$

where:

$$\begin{cases} r_+ = \dfrac{1}{2}\sqrt{(x_c + y_s)^2 + (y_c - x_s)^2} \\ r_- = \dfrac{1}{2}\sqrt{(x_c - y_s)^2 + (y_c + x_s)^2} \end{cases} \tag{28}$$

The type of precession is given by the sign of B so:

B>0 – precession is direct;
B<0 – precession is inverse;
B=0 – precession is linear.

4. Conclusions

1. Mathematical modeling of hydropower equipment vibration phenomena requires, in essence, determining the dynamic response of rotor-bearing assembly in the structure of HPA, when exercising perturbations on them.
2. Mathematical modeling of vibration phenomena (sources impact, propagation in rotor-shaft assembly,) at HPA is very laborious, impossible to apply in the general case.
3. A simpler version of solving the problem of dynamic response of a HPA is to use finite element method, whose main advantage is that it allows a relatively easy modeling of a complex system such as HPS, and also a possibility to assess the gyroscopic effect, the effect of the shearing force and of the axial one, of torsional moments, of the distortion of the shaft and of the hydrodynamic force on the vibration levels.
4. Applying the finite element method at the analysis of vibrational processes of the HPA requires the following steps: setting up the associated network, writing the equations and assembling them, solving the system of equations, determination and interpretation of the response.
6. For modeling of a rotor-bearing unit of a HPA, after establishing the axis system, in order to determine the kinetic energy of the rotor, of the shaft, of the potential shaft distorting energy and of the generalized forces in the bearings, it is necessary to use Lagrange's equations, so that motion equations obtained in the form of matrices, lead by resolution, to the determination of the eigenmodes of vibration, of the loads and reactions in the bearings.
7. Solving the assessment model of vibrations of a HPA requires the use of appropriate software, an example being the computer program, elaborated for this purpose, in the MATLAB environment.

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