ACTIVE POWER FLUCTUATIONS IN CIRCUITS WITH CONCENTRATED AND DISTRIBUTED PARAMETERS UNDER TRANSIENT REGIME

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Abstract - This paper examines an analytical method for calculation of voltage, current and instantaneous active power evolution in DC circuits with distributed and concentrated parameters in dynamic regimes. As a reactive power sources there are considerers capacitor without electrical load and an inductor connected to power line. The relations of exact analytical solution for calculation of instantaneous values of voltage and current, as well as the instantaneous value of the active and reactive powers have been obtained using the method of characteristics. The obtained relations take into account the physical essence of voltage and current wave propagation in inhomogeneous long line. Parametric analysis of the transient features has been performed using relative units system, which provides the general representation of the results. It is demonstrated, that for transient regimes there are possible the strong fluctuations of the active power values absorbed by pure active load of the line depending on the capacitance of the reactive power source (RPS) and on the connection place. The amplitude of these fluctuations exceeds more than 3 times the amplitude values of the absorbed power in steady state regime. The connection of an inductor in the line reduces the fluctuation phenomena of the absorbed power in transient mode, as well as the own line losses. The observed peculiarities can be taken into account when adjusting the protection systems of DC lines and developing measures (if necessary) for limitation of the active power fluctuations in long lines under the transient regimes.

Keywords: active power, fluctuation, inhomogeneous line, transient process, exact analytical solution.

1. INTRODUCTION

The different operating modes, in which the phenomena of dissipation, dispersion and the exchange of energy between the generator and the load and between the different parts of the circuit is persistent, are possible. The computational methods and analysis of the mentioned processes for stationary regimes are well developed [5]. In case of transient modes, these processes are more complex and their study often is facing with different difficulties conditioned by the fact, that the conditions of applicability of the methods are not always clear, as it is robust for steady state regimes [1-3]. It may be mentioned, that up to now there is no some entirely recognized by the scientific community treatment for the problem that relates to the study of active and reactive power in transient modes in the case of the sinusoidal current and voltage curves deviation from a sinusoidal form, for example, for deforming and pulse regimes [1].

2. INVESTIGATION PROBLEM

Let it be required to perform the exact quantitative analysis of the evolution of unsteady currents and voltages for short time intervals (ms) in order to determine the conditions of attainment of the maximum values of the instantaneous power absorbed by the load (pulse mode load). In order to obtain the exact solution concerning particularities of these regimes, we examine the dynamic processes that take place when connecting to DC line of the passive dipoles formed by the capacitor without electrical load or by inductor with concentrated parameters, that present the reactive power sources (SPR) for the line. The processes in circuits of this type can be described by telegraph equations [6], and the method of their analysis should take into account the in homogeneities, that are included in the line with distributed parameters.

The aim is to obtain the exact solution for determining the voltage and the current in the long line with losses in transient regime, the evolution of the active and the reactive instantaneous powers of the generator and the load by the method of characteristics [7,8] for the line without signal distortion: $R / L = G / C = \gamma$.

3. EXACT ANALYTICAL SOLUTION

Let examine the line with losses (l is its length) connected to the DC voltage source U_0 that supplies the active load R_S. We suppose that the capacitor without electrical load C_n (uncharged capacitor) is momentary connected at the point $x_n = l/2$ in the time moment $t = t^* > 0$ (fig. 1). The problem will be examined in more consecutive steps concerning the calculation of quantities that characterize the dynamic processes in inhomogeneous line with distributed parameters and in lines with concentrated parameters.

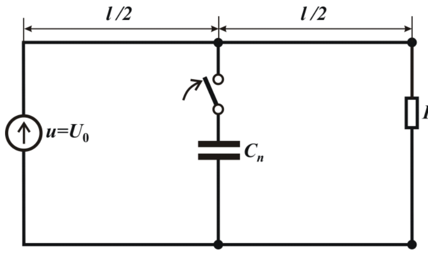

Fig. 1. Uncharged capacitor connection to the DC line

To determine the voltage function $u(x,t)$ and current function $i(x,t)$ in the circuit from fig.1 we consider the system of telegraph equations

$$L\frac{\partial i}{\partial t}+\frac{\partial u}{\partial x}+Ri=0;\ C\frac{\partial u}{\partial t}+\frac{\partial i}{\partial x}+Gu=0,\ \ x\in[0,l],\, t>0 \quad (1)$$

and the following initial and boundary conditions:

$$u(x,0)=i(x,0)=0,\ \ x\in[0,l], \quad (2)$$

$$u(0,t)=U_0,\ \ u(l,t)=R_S i(l,t),\ \ t\ge 0, \quad (3)$$

$$C_n\frac{du(x_n,t)}{dt}=i(x_n-0,t)-i(x_n+0,t), \quad (4)$$

can be $t>t^*;\ u(x_n,t^*)=0$

The exact solution for the formulated problem (1) – (4) ca be obtained for the line without signal distortion $R/L=G/C=\gamma$ by the method of characteristics [7, 8]. According to this method till the time moment t^* the solution is determined in [7] and it has the following form

$$u_{2n}(t)=U_0;\ \ i_{2n}(t)=\frac{U_0}{Z_B}\left[\ 1+2\sum_{j=1}^{n-1}(-z_\gamma)^j\ \right]=\frac{U_0\left[\ 1-z_\gamma-2(-z_\gamma)^n\ \right]}{Z_B(1+z_\gamma)} \quad (5)$$

when $x=0$, $t\in[2(n-1)\Delta,2n\Delta],\ n=1,2,3,...;$

$$u_{2n+1}(t)=(1+z)U_0e^{-\gamma\Delta}\sum_{j=0}^{n-1}(-z_\gamma)^j=(1+z)U_0e^{-\gamma\Delta}\frac{1-(-z_\gamma)^n}{1+z_\gamma},$$

$$i_{2n+1}(t)=\frac{(1-z)U_0e^{-\gamma\Delta}}{Z_B}\sum_{j=0}^{n-1}(-z_\gamma)^j=\frac{(1-z)U_0e^{-\gamma\Delta}}{Z_B}\frac{1-(-z_\gamma)^n}{1+z_\gamma} \quad (6)$$

when $x=l$, $t\in[(2n-1)\Delta,(2n+1)\Delta],\ n=1,2,3,....$

Here in (5) and (6) we use the following notations:

$$z=\frac{R_S-Z_B}{R_S+Z_B},\ z_\gamma=ze^{-2\gamma\Delta},\ Z_B=\sqrt{\frac{L}{C}},\ \Delta=l/a,\ a=\frac{1}{\sqrt{LC}}. \quad (7)$$

Let's suppose that the uncharged capacitor is connected to the middle point of the line in the time moment $t^*=2n^*\Delta$. Then from the correlation on the characteristics $x\pm at=const$ for $t\in[t^*,t^*+\Delta/2]$ at the connection point $x_n=l/2$ we obtain

$$u(x_n-0,t)+Z_Bi(x_n-0,t)=$$
$$=\left[\ u(0,t-\Delta/2)+Z_Bi(0,t-\Delta/2)\ \right]e^{-\gamma\Delta/2}=$$
$$=\left[\ U_0+Z_Bi_{2n^*}\ \right]e^{-\gamma\Delta/2}\equiv A_0.$$
$$u(x_n+0,t)-Z_Bi(x_n+0,t)=$$
$$=\left[\ u(l,t-\Delta/2)-Z_Bi(l,t-\Delta/2)\ \right]e^{-\gamma\Delta/2}=$$
$$=\left[\ u_{2n^*+1}-Z_Bi_{2n^*+1}\ \right]e^{-\gamma\Delta/2}\equiv B_0. \quad (8)$$

Then we add to these relations the voltage continuity condition and the differential equation (4):

$$u_n=u(x_n-0,t)=u(x_n+0,t),$$

$$C_n\frac{du_n}{dt}=i_n,\ \ i_n=i(x_n-0,t)-i(x_n+0,t). \quad (9)$$

The exact solution of the equations (8) – (9) with regard to voltage and current values in the point x_n is the following

$$u_{n,0}(t)=\frac{A_0+B_0}{2}\left[\ 1-e^{\lambda_n(t-t^*)}\ \right],\ i_{n,0-}(t)=\frac{A_0-u_{n,0}(t)}{Z_B},$$

$$i_{n,0+}(t)=\frac{u_{n,0}(t)-B_0}{Z_B},\ i_{n,0}(t)=i_{n,0-}(t)-i_{n,0+}(t),$$

$$\lambda_n=-\frac{2}{C_nZ_B},\quad x=x_n,\, t\in[t^*,t^*+\Delta/2]. \quad (10)$$

4. COMPUTATIONAL ALGORITHM

The algorithm of obtaining the solution by the method of characteristics consists of the following. In the highlighted points, the values of voltage and current are modified conceptually only when the incident or reflected wave is coming. These modifications occur in discrete time intervals, which are determined by the length of the parts of the line and by the speed of propagation of the potential and current waves in the line. These discrete intervals can be described by the relation: $t\in[t^*+m\Delta/2,t^*+(m+1)\Delta/2]$, m =0,1, 2, 3, …, where

m is the serial number of the wave race in the propagation process in the examined section and Δ is the duration of the electromagnetic wave propagation in line with length l.

During the time interval $t \in [t^*, t^* + \Delta/2]$ at the end points of the line the solution remains as (5) – (6):

$$u_{0,0} = u(0,t) = U_0,\ i_{0,0} = i(0,t) = i_{2(n^*+1)},$$

$$x = 0, t \in [t^*, t^* + \Delta/2]; \quad (11)$$

$$u_{l,0} = u(l,t) = u_{2n^*+1}, i_{l,0} = i(l,t) = i_{2n^*+1},$$

$$x = l, t \in [t^*, t^* + \Delta/2], \quad (12)$$

because the perturbations caused by capacitor connection to the line does not reach the ends of the line, i.e. the points $x = 0$ and $x = l$.

During the next time interval $t \in [t^* + \Delta/2, t^* + \Delta]$ the wave from capacitor reaches the end points of the line and the solution for the points $x = 0$ and $x = l$ becomes as follows:

$$u_{0,1}(t) - Z_B i_{0,1}(t) = \\ = \left[u_{n,0}(t - \Delta/2) - Z_B i_{n,0-}(t - \Delta/2) \right] e^{-\gamma\Delta/2} \equiv D_1(t), \\ u_{0,1}(t) = U_0,\ i_{0,1}(t) = \left[U_0 - D_1(t) \right] / Z_B,\ x = 0; \quad (13)$$

$$u_{l,1}(t) + Z_B i_{l,1}(t) = \\ = \left[u_{n,0}(t - \Delta/2) + Z_B i_{n,0+}(t - \Delta/2) \right] e^{-\gamma\Delta/2} \equiv E_1(t),$$

$$u_{l,1}(t) = R_S i_{l,1}(t),\ u_{l,1}(t) = (1+z) E_1(t)/2, \\ i_{l,1}(t) = (1-z) E_1(t)/(2Z_B),\ x = l \quad (14)$$

During the same time interval $t \in [t^* + \Delta/2, t^* + \Delta]$ at the middle point of the line $x = l/2$ we have

$$A_1(t) = \left[U_0 + Z_B i_{0,0}(t - \Delta/2) \right] e^{-\gamma\Delta/2},$$

$$B_1(t) = \left[u_{l,0}(t - \Delta/2) - Z_B i_{l,0}(t - \Delta/2) \right] e^{-\gamma\Delta/2},$$

$$u_{n,1}(t) = u_{n,0}(t^* + \Delta/2) e^{\lambda_n (t - t^* - \Delta/2)} - \\ -\frac{\lambda_n}{2} \int_{t^*+\Delta/2}^{t} \left[A_1(\tau) + B_1(\tau) \right] e^{\lambda_n (t-\tau)} d\tau,\ \lambda_n = -\frac{2}{C_n Z_B},$$

$$i_{n,1-}(t) = \frac{A_1(t) - u_{n,1}(t)}{Z_B},\ i_{n,1+}(t) = \frac{u_{n,1}(t) - B_1(t)}{Z_B},$$

$$i_{n,1}(t) = i_{n,1-}(t) - i_{n,1+}(t),\ x = x_n, t \in [t^* + \Delta/2, t^* + \Delta]. \quad (15)$$

During the next time interval $t \in [t^* + \Delta, t^* + 3\Delta/2]$ the solution at the end points of the line takes the form

$$u_{0,2}(t) - Z_B i_{0,2}(t) = \\ = \left[u_{n,1}(t - \Delta/2) - Z_B i_{n,1-}(t - \Delta/2) \right] e^{-\gamma\Delta/2} \equiv D_2(t),$$

$$u_{0,2}(t) = U_0,\ i_{0,2}(t) = \left[U_0 - D_2(t) \right] / Z_B,\ x = 0; \quad (16)$$

$$u_{l,2}(t) + Z_B i_{l,2}(t) = \\ = \left[u_{n,1}(t - \Delta/2) + Z_B i_{n,1+}(t - \Delta/2) \right] e^{-\gamma\Delta/2} \equiv E_2(t), \\ u_{l,2}(t) = R_S i_{l,2}(t),\ u_{l,2}(t) = (1+z) E_2(t)/2 \\ i_{l,2}(t) = (1-z) E_2(t)/(2Z_B),\ x = l. \quad (17)$$

During the same interval $t \in [t^* + \Delta, t^* + 3\Delta/2]$ at the middle point of the line we obtain for voltage and current the following formulas:

$$A_2(t) = \left[U_0 + Z_B i_{0,1}(t - \Delta/2) \right] e^{-\gamma\Delta/2},$$

$$B_2(t) = \left[u_{l,1}(t - \Delta/2) - Z_B i_{l,1}(t - \Delta/2) \right] e^{-\gamma\Delta/2},$$

$$u_{n,2}(t) = u_{n,1}(t^* + \Delta) e^{\lambda_n (t - t^* - \Delta)} - \\ -\frac{\lambda_n}{2} \int_{t^*+\Delta}^{t} \left[A_2(\tau) + B_2(\tau) \right] e^{\lambda_n (t-\tau)} d\tau,$$

$$i_{n,2-}(t) = \frac{A_2(t) - u_{n,2}(t)}{Z_B},\ i_{n,2+}(t) = \frac{u_{n,2}(t) - B_2(t)}{Z_B},$$

$$i_{n,2}(t) = i_{n,2-}(t) - i_{n,2+}(t),\quad x = x_n, t \in [t^* + \Delta, t^* + 3\Delta/2]. \quad (18)$$

In this way the described algorithm allows to obtain the exact analytical solution of the problem for any time interval $t \in [t^* + m\Delta/2, t^* + (m+1)\Delta/2]$, $m = 0, 1, 2, \ldots$ So we get at the ends of the line the following generalized computational expressions for voltage and current functions:

$$u_{0,m}(t) - Z_B i_{0,m}(t) = \\ \left[u_{n,m-1}(t - \Delta/2) - Z_B i_{n,(m-1)-}(t - \Delta/2) \right] e^{-\gamma\Delta/2} \equiv D_m(t),$$

$$u_{0,m}(t) = U_0,\ i_{0,m}(t) = \left[U_0 - D_m(t) \right] / Z_B,\ x = 0; \quad (19)$$

$$u_{l,m}(t) + Z_B i_{l,m}(t) =$$

$$\left[u_{n,m-1}(t - \Delta/2) + Z_B i_{n,(m-1)+}(t - \Delta/2) \right] e^{-\gamma\Delta/2} \equiv E_m(t),$$

$$u_{l,m}(t) = R_S i_{l,m}(t),\ u_{l,m}(t) = (1+z) E_m(t)/2,$$

$$i_{l,m}(t) = (1-z) E_m(t)/(2Z_B),\ x = l \quad (20)$$

and at the middle point of the line we have the following relations

$$A_m(t) = \left[U_0 + Z_B i_{0,m-1}(t - \Delta/2) \right] e^{-\gamma\Delta/2},$$

$$B_m(t) = \left[\ u_{l,m-1}(t-\Delta/2) - Z_B i_{l,m-1}(t-\Delta/2)\ \right] e^{-\gamma\Delta/2},$$

$$u_{n,m}(t) = u_{n,m-1}(t^* + m\Delta/2) e^{\lambda_n (t - t^* - m\Delta/2)} -$$
$$-\frac{\lambda_n}{2} \int\limits_{t^*+m\Delta/2}^{t} \left[\ A_m(\tau) + B_m(\tau)\ \right] e^{\lambda_n (t-\tau)} d\tau,$$

$$i_{n,m-}(t) = \frac{A_m(t) - u_{n,m}(t)}{Z_B},\ i_{n,m+}(t) = \frac{u_{n,m}(t) - B_m(t)}{Z_B},$$

$$i_{n,m}(t) = i_{n,m-}(t) - i_{n,m+}(t),$$

$$x = x_n, t \in [t^* + m\Delta/2, t^* + (m+1)\Delta/2]. \qquad (21)$$

The obtained calculation formulas allow us to study the dynamic processes in electrical lines and to execute the extensive parametric analysis of the operating mode.

5. PARAMETRIC ANALYSIS OF ENERGY PROCESSES IN INHOMOGENEOUS LONG LINE

Let's study at first the wave processes in the ideal line ($R = G = 0$), that has as a load the active resistance adjusted to the line parameters $R_S = Z_B = 1$, for example, the influence of reactive loads on the development of the instantaneous power or of the impulse power. Such regimes may be characteristic for testing the equipment and machinery used in experimental research in physics, for achieving the electrotechnologies in processing the materials, etc. The connection of parameters and ensuring the transmission of the maximum possible active power for this circuit helps to reduce the cost of the necessary equipment for research and experimental research expenditures. The specific regimes of this kind can be observed even in DC networks, that impose necessity to know the peculiarities of transient processes running in these circuits, including the section, that refers to adjustment of protection systems with the aim of excluding unjustified (false) disconnections of networks.

For convenience, the values of the analyzed circuit parameters are given in relative units (r.u.) in accordance with the recommendations from [7].

The connection of the line with the adjusted load $R_S = Z_B = 1$ to the power supply is followed by the establishment of instantaneous steady state with the unit values of current, voltage, power injected by the source and power absorbed by the load (r.u.). Let's suppose now that the uncharged capacitor C_n is momentary connected at the point $x_n = l_n = l/2$ in the time moment t = 0.3125 corresponding to the 5 wave runs along the line with the equivalent length $l = 1/16$ of the wavelength of the signal with frequency of 50 Hz.

The dynamics of the power changes of the generator, loading and reactive power source (RPS) is represented in the fig. 2 (curves 1-3) with consecutive doubling of the value C_n = 1(*a*); 2(*b*); 4(*c*); 8(*d*). The increasing of this value is accompanied by the disproportionate but monotonous increasing of the pure active loading power during the transient process, so we have the following set of values for the power absorbed by the active load: P_1 = 3.13; 3.33; 3.50; 3.64, correspondingly.

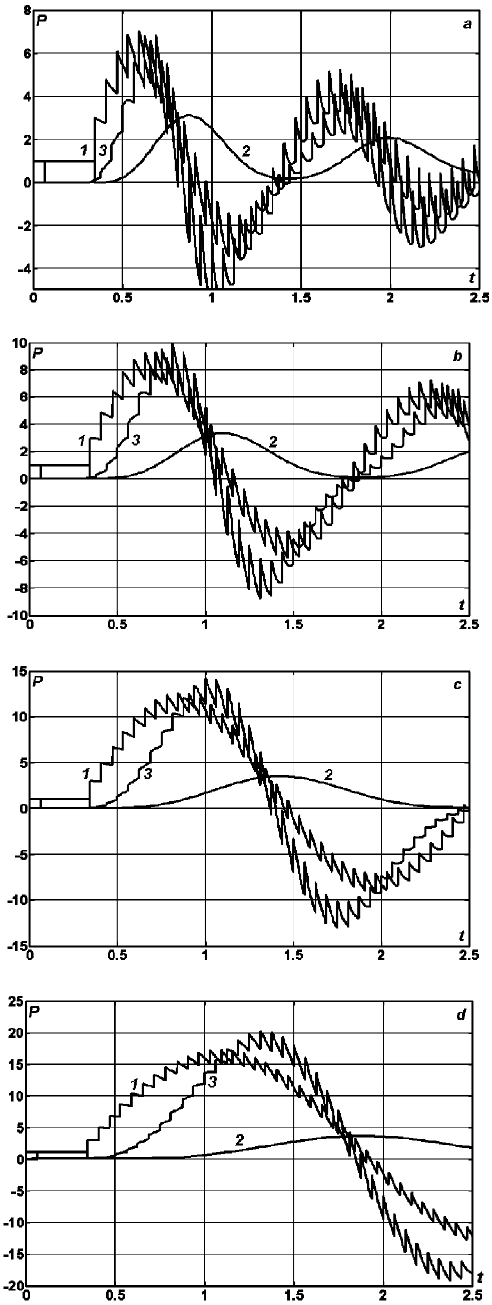

Fig. 2. The instantaneous powers of the generator, loading and RPS (curves 1-3), when $R = G = 0$; C_n = 1(*a*); 2(*b*); 4(*c*); 8(*d*), where time and power are presented in relative units

Taking into account the losses in the continuous current line (fig. 3) we observe some decreasing of power rushes and nonlinear dependence of its maximal values on the values of the connected capacity: P_1 = 2.64; 2.68; 2.63; 2.5.

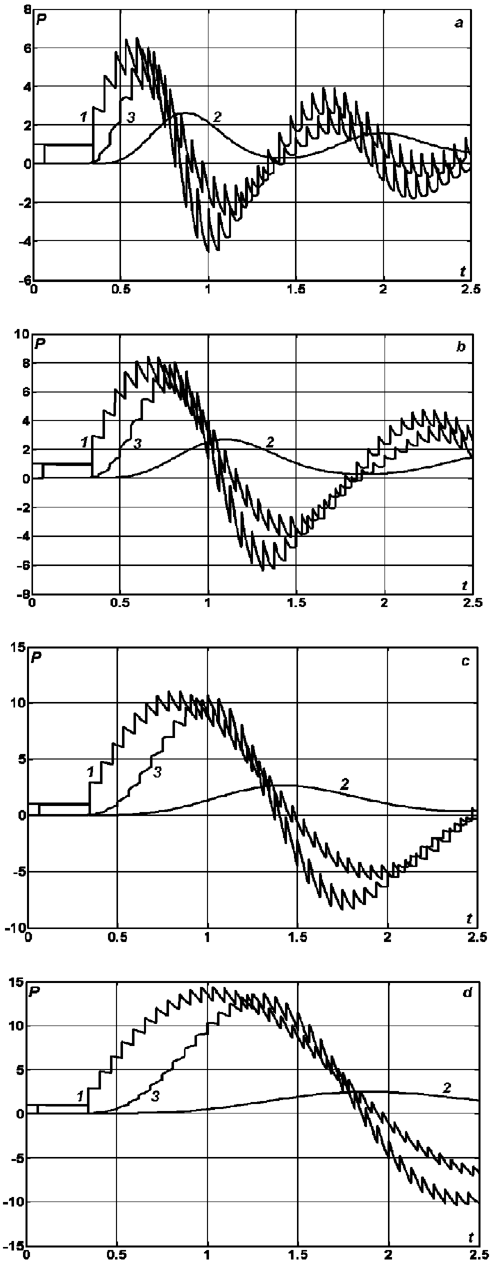

Fig. 3. The instantaneous powers of the generator, loading and RPS (curves 1-3), when $R = 7G = 0.48$; $C_n = 1(a);2(b);4(c);8(d)$, where time and power are presented in relative units

On the basis of these calculations it may be established the following. Thus, as a first approximation the value C_n = 2 can be considered the optimal one in respect to reaching the ultimate loading power under the impulse mode and with capacitor connection at the middle point of the continuous current line.

If the capacitor C_n = 2 is connected closer to the source, for example at the distance $l_n = l/4$, then we get some increasing of the loading power: P_1 = 2.95 (fig. 4, *a*). However, the parametrical analysis makes it clear that the amplitude of oscillation of the active power can be raised no more than three-fold relatively to nominal level. It is interesting to mention, that these fluctuations can be eliminated, if the inductance L_m is connected to the line concomitantly with the capacitor. It is significant that the location of its series connection ($x_m = l_m$) has no influence on this effect (fig. 4, *b*-*d*).

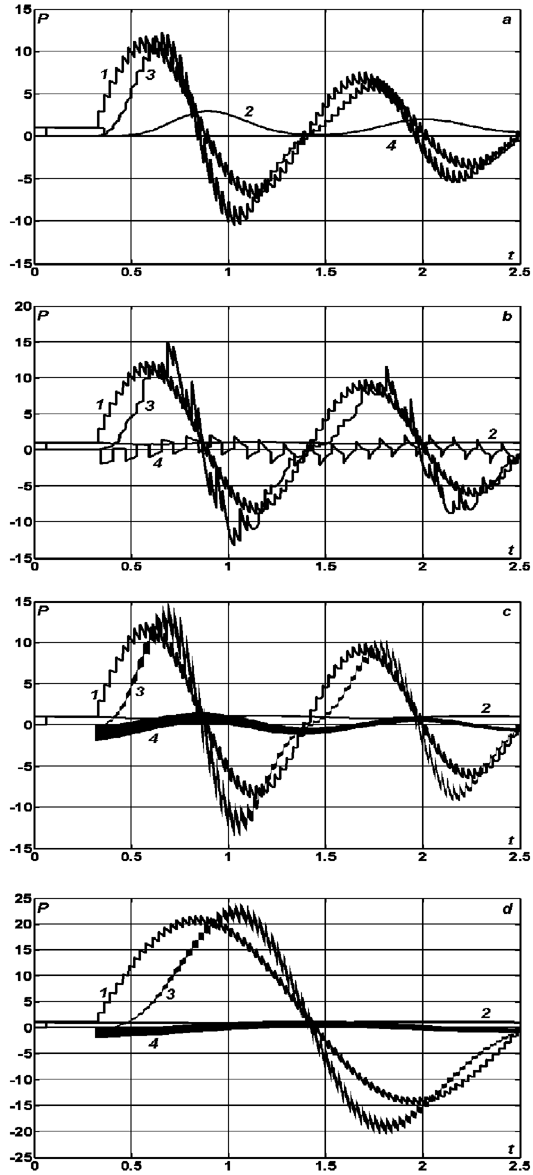

Fig.4. The instantaneous powers of the generator, loading and RPS (curves 1-4) when $R = 7G = 0.48$; $C_n = 2$, $l_n = l/4$ (*a*); $C_n = L_m = 2$, $l_n = l/4$, $l_m = 3l/4$ (*b*); $C_n = L_m = 2$, $l_n = l_m = l/4$(*c*); $C_n = L_m = 8$, $l_n = l_m = l/4$(*d*, where time and power are presented in relative units

6. CONCLUSIONS

- The exact analytical solutions for voltage and current functions at the specified points of inhomogeneous long line have been obtained by the method of characteristics. The obtained relations allow calculation of instantaneous active and reactive power, the evolution of the reactive power of the condenser connected to the circuit line.

- The computational algorithm has been proposed for calculating the exact solutions for both the ideal line (without losses) and for the line with losses. Line without signal distortion appears as a particular case of the line with losses.
- Parametric analysis of the dynamic processes made it possible to detect the fluctuations of the active power of the generator and the load in the lines conditioned by propagation phenomenon and the influence of reactive power source on the absorption capacity of the active power during transient regimes.
- The own energy losses in the DC line has the influence on the absorption capacity of the active power of the load during the transient mode. This capacity is reduced compared to the ideal line (without losses).
- The absorption capacity of the active power of the load during the transient mode is influenced by the location of the RPS connection in relation to the load. When approaching to the load this capacity increases. Fluctuations of the instantaneous active power load in transient regimes in DC line can be excluded if the inductance is connected to the line concomitantly with the capacitor.

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