THE SOLUTION OF TELEGRAPHIC EQUATIONS LIVE AND RETURN TIME

ERHAN, F*, RIMSCHI, V**., NANTOI, O***.,
*Agrarian State University of Moldova, MD-2049, str. Mircesti,56, Chisnau;
** Power Energy Institute of Academia of Scientific Moldova; MD-2072,str.Academiei,5, Chisnau;
*** Institute of Programme
terhan@mail.ru

Abstract. The article is devoted to the analytical solution of the telegraph equations for a semi-infinite line with losses with an arbitrary form of input voltage. An exact analytical solution for a semi-infinite line with losses with an arbitrary input voltage form is proposed as model.

The results of model calculations are given, which clearly illustrate the lack of alternatives to PaPuRi, an algorithm presented in the form of a computer program in the Matlab application environment.

Keywords: lightning conductor, semi-infinite line, telegraph equations, reference solution, PaPuRi – algorithm

1. INTRODUCTION

It is known that under the notion of lightning is meant a long line with the terminals connected to the earth socket, with the purpose of protecting the electrical equipment from the surges which may appear in the electrical equipment as a result of the atmospheric phenomena's may occur.

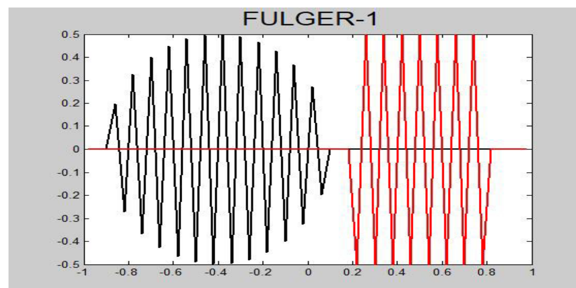
Any three-phase consumer feedstock is one of the basic inventions in the nineteenth-century electrotehnical field, and is most often protected from atmospheric overvoltage by lightning strike.

It can be noticed that until now in the specialized textbooks and technical literature physical-mathematical models which would adequately describe the dynamics of the electromagnetic impulses, or in the phase breaking and its connection to the earth are missing.

2. CALCULATION METHODS

In the compartments devoted to the analytical calculations of non-stationary processes and waves in long lines [1-9] are given, only the analytical computing technologies that are outdated and correspond only to the fourth stage of analytical computing process development corresponding to the 20th century , which is based on the extensive use of logarithmic ruler, compass and arithmeters, and none of the PCs and artificial intellect.

The electric currents created in this way in the repeating lines by the lightning strike in the lightning conductor in the first moment can have amplitude of tens of kA, and the shape of both the quench and the respective voltages can have different shapes that can coincide with the shapes of (Fig. 1).



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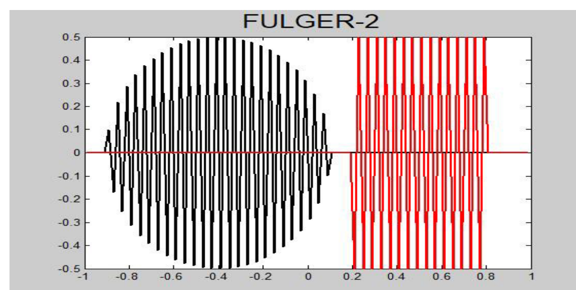

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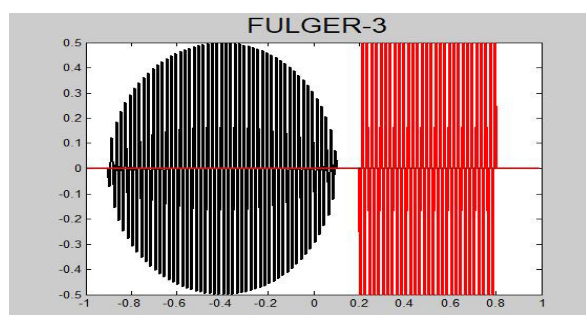

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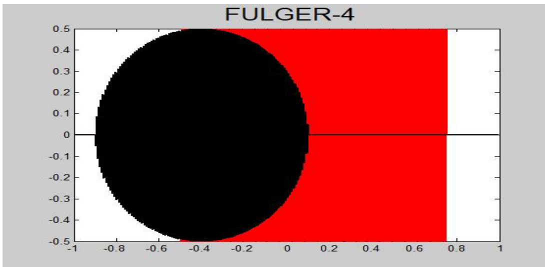

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Fig. 1. The forms of electric currents and voltages that may occur in the lines of lightning after lightning strikes in the lightning of that line.

With time, the electromagnetic waves attenuate quite quickly, practically until their disappearance, because the amplitudes of the respective waves decrease to a rather low value reaching the limits $(10^{-50} \le I_m \le 10^{-305})A$.

In this case, does the question arise, can it be restored after the respective final values of the electrical current exactly its initial value within the tolerable error limits, for the time $(\Delta\tau = 0)$ being aware of the final values ?

According to [9], it can be observed that the treatment of such problems can be reported to the limit problems for the reversal time to the incorrect problems due to the instability of the initial information.

In spite of all these phenomena, the PaPuRi algorithm [10] successfully ended with the solution of problems of this kind, which at first seemed unlikely to have analytical solutions.

This is how to solve the problems of this type, it is proposed to familiarize all the people in the given article.

According to [3,5], there are presented the exact analytical solutions of the unstable transient phenomena of spreading the rectangular electromagnetic waves in semi-finite lines having the following parameters: $L = C = 1; R = 2; G = 0$.

In this case the initial telegraphic equations are presented in the form of the variable values according to the analytical relation (1).

$$L\frac{\partial i}{\partial t} + \frac{\partial u}{\partial t} + Ri = 0$$
$$C\frac{\partial u}{\partial t} + \frac{\partial i}{\partial t} + Gu = 0 \qquad (1)$$

If the equation (1) will be integrated based on the initial conditions, so in the fulfillment of conditions $u(x,0) = i(x,0) = 0$ and with the condition of the voltage drop at the beginning of the line (at the input), then the relation of the form (2) will be fulfilled.

$$u(0,t) = 1 \qquad (2)$$

For this reason, the momentary connection of the respective line to a constant voltage can be prearranged and analyzed with a null approximation in order to construct a lightning pattern.

The distribution of the current in such an analytical model both in time and space is presented [1, 2], in a compact form according to the analytical relation (3).

$$i(x,t) = I_0\left(\frac{R}{2}\sqrt{t^2 - x^2}\right) \cdot e^{-\frac{Rt}{2}} ;\text{if } t \succ x; i(x,t) = 0, \qquad (3)$$

where: - $I_v(z) = I_v(jz)$ represents the Bessel function of the first order of v.

If we take into account the equalities of the leaps of the sought functions, then the equation of form (4) is fulfilled at the wave front:

$$\lim_{\varepsilon \succ 0, \varepsilon \to 0} u(x, x+\varepsilon) = \lim_{\varepsilon \succ 0, \varepsilon \to 0} i(x, x+\varepsilon) = e^{-\frac{Rt}{2}} \qquad (4)$$

In this case, according to [3,4] the analytical expression for the function differentiated by the function value (x) for determining the electric current will be determined from the relation (5).

$$\frac{\partial i(x,t)}{\partial x} = \frac{R \cdot x \cdot e^{-\frac{Rt}{2}}}{2\sqrt{t^2 - x^2}} I_1\left(\frac{R}{2}\sqrt{t^2 - x^2}\right) \qquad (5)$$

For the analyzed variant the analytical value of the voltage according to the relation (6) will be obtained:

$$u(x,t) = e^{-\frac{Rt}{2}} - \frac{R}{2}\int_x^t \frac{x \cdot e^{-\frac{Rz}{2}}}{\sqrt{z^2 - x^2}} I_1\left(\frac{R}{2}\sqrt{z^2 - x^2}\right) dz;$$

$$\text{if } t \succ x; \text{ și } u(x,t) = 0 \text{ ; if } t \prec x; \qquad (6)$$

If the homogeneous and linear values of the partial-type telegraph equations of type (1) are taken into account, then the analytical relations of type (4) and (6) can be integrated for any arbitrary value of the input voltage $u(0,t) = f(t)$ according to the relation (7- 8):

$$i(x,t) = f(t-x) \cdot e^{-\frac{Rx}{2}} - \int_0^{t-x} \frac{\partial i_1}{\partial t}(x, t-z) f(z) dz \text{ ; if } t \succ x; \qquad (7)$$

$$u(x,t) = f(t-x) \cdot e^{-\frac{Rx}{2}} - \int_0^{t-x} \frac{\partial u_1}{\partial t}(x, t-z) f(z) dz;$$

$$\text{if } t \succ x; \text{ and } i(x,t) = u(x,t) = 0; \text{ if } t \prec x; \qquad (8)$$

where:

$$\frac{\partial u_1(x,t)}{\partial t} = \frac{R \cdot x \cdot e^{-\frac{Rt}{2}}}{2\sqrt{t^2 - x^2}} I_1\left(\frac{R}{2}\sqrt{t^2 - x^2}\right) \cdot e^{-\frac{Rt}{2}} \text{ ; if } t \succ x;$$

$$\frac{\partial i_1(x,t)}{\partial t} = \frac{R \cdot x \cdot e^{-\frac{Rt}{2}}}{2\sqrt{t^2 - x^2}} I_1\left(\frac{R}{2}\sqrt{t^2 - x^2}\right) + e^{-\frac{Rt}{2}} I_0\left(\frac{R}{2}\sqrt{t^2 - x^2}\right) \text{; if } t \succ x;$$

$$\frac{\partial u_1(x,t)}{\partial t} = \frac{\partial i_1(x,t)}{\partial t} = 0; \text{ if } t \prec x;$$

It can be seen that the presence in the above-mentioned expressions of the integrals having the variable upper limit must be numerically determined, and the integral solutions for the leakages that differ from zero ($G \succ 0$) (except for the case where the line does not have of distortions: $G = R$), or for lines having a finite length can be presented as complex (imaginary) values.

For such cases, they can be quadrature solutions and can be built for some particular cases, but such cases may have quite bulky solutions and virtually impossible to use to solve them through computers.

It is quite difficult to imagine how the analytical expressions will be used to determine the circumference of the waveforms in the lines of a determined length, after a few dozen reflections at the end, or from the point of connection of two lines or other non-uniformities.

It is necessary to mention that for the case of an ideal line that does not lose losses $(G = R = 0)$ with an unlimited length the generator current, resulting from the exact real solution of the relation of the form (7) is determined according to the relation (9) for any moment of time ($t \succ x;$).

$$i(0,t) = u(0,t)/z_u \quad (9)$$

where: z_u - is the wave impedance of the analyzed line and (long line) determined according to [3-6] of the relationship (10).

$$z_u = \sqrt{L/C} \quad (10)$$

where: L-is the logic inductance of the contour formed by the direct and reverse conductor, (Hn / m);

C - is the cross-sectional capacity between the contour formed by the direct and the reverse conductor, (F / m).

If the value of the electrical current function is given at the input of the long line, then the line voltage at the end of the line shall be determined according to the relationship (11).

$$u(0,t) = i(0,t) \cdot z_u . \quad (11)$$

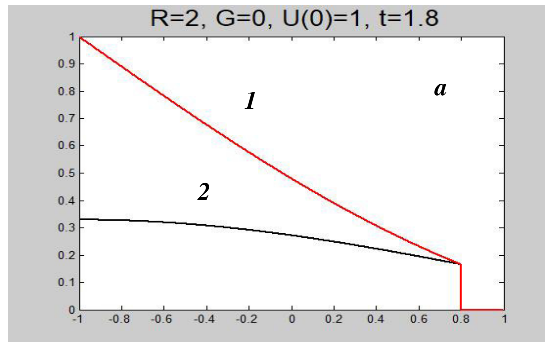
As can be seen from the exact solution of the expression (7) in real electrical loops that have losses, that is $(G \neq R \neq 0)$, the relation is fulfilled only to the direct electromagnetic wave that is in the direction of the primary front.

Because the possibilities of the analytical solutions are quite limited, which does not allow to surpass the limits of the fourth technological level of computation, its significance is very difficult to appreciate, because the solutions obtained by means of the dispersion lines are only the ones with which can be used as a test in the process of assessing the apostatical exactitude of numerical computational methods and computerized information technology (IT) by five or six times.

From the real practice of using the real schemes for the analytical drill, for the solution of the differential equations with partial derivations, the preventive appreciation of the calculations (the order of approximation, the stability and the hardness of the solutions ensuring the convergence to the unlimited shredding of the network step) by a practical tangential solution does not by [10].

At the beginning of the calculation, the function limit $u(0,t) = f(t)$ $u(0,t) = f(t)$ is given in the form of a series of 7 (seven) stages (single - pole impulses of rectangular shape) deviating from each other with a determined period of time equal to the value $(\Delta\tau = 0,1)$, and $(f(t) \prec 1,4)$.

If the equivalent transformation of the relation (7) is performed in a set of uniform n = 2000 ranges within the node variation limits $(-1,0 \le x \le +1,0)$ for the time $(t = 1,8)$ point and its tabulation, then the analytical results of the respective calculations are presented on (fig.2).



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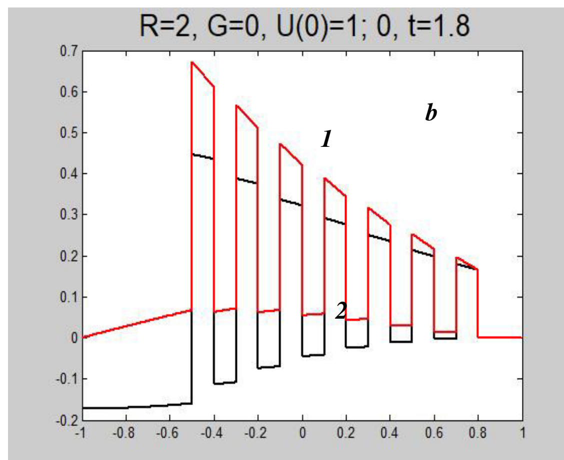

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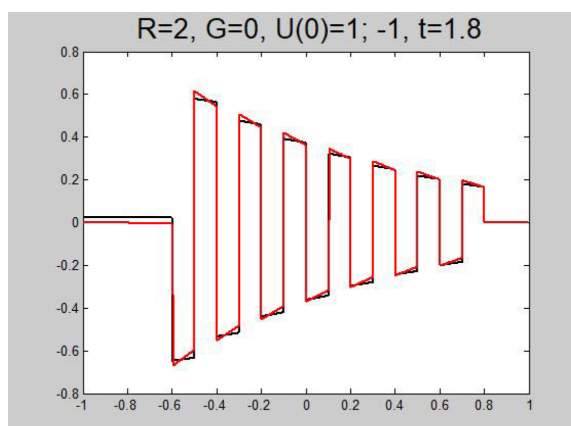

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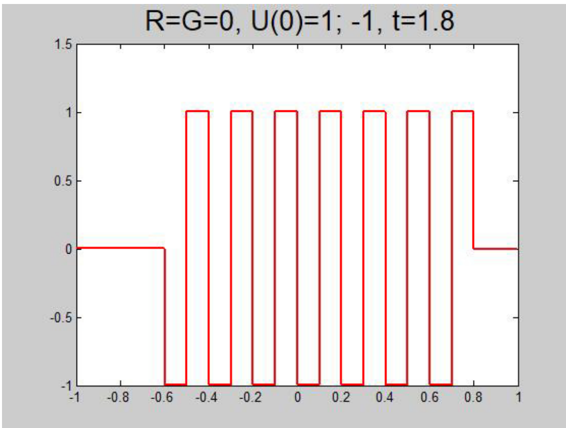


d

Fig. 2. The distribution of voltage (curve -1) and current (curve-2) in the analyzed line at time t = 1.8 at monopolar impulses (a, b) and bipolar pulses (c, d).

For this time of frontal time the direct electromagnetic wave that spreads from source to consumer with a unitary speed proves to cross the line length $(l = 1,8)$.

In the Matlab computing environment, the function $f(t)$ will be presented as

$$f(t) = (j \prec 1400)\cdot((-1)floor(j/100))/2$$

(a multipolar impulse) if

$$f(t) = (j \prec 1400)\cdot((-1)floor(j/100))/2$$

a bipolar impulse.

The results of the analytical calculations are shown in (fig.2, a, b) for a line segment when the conditions of form (12) are fulfilled.

$$h = \tau = 0,001; L = C = 1; z_u = \sqrt{L/C = 1}\,. \qquad (12)$$

For lines without loss $(R = G = 0)$ or without distortion $(R = G)$, then in such lines for any form of the inlet manifold the relation of form (13) is fulfilled:

$$u(t) = z_u \cdot i(t)\,. \qquad (13)$$

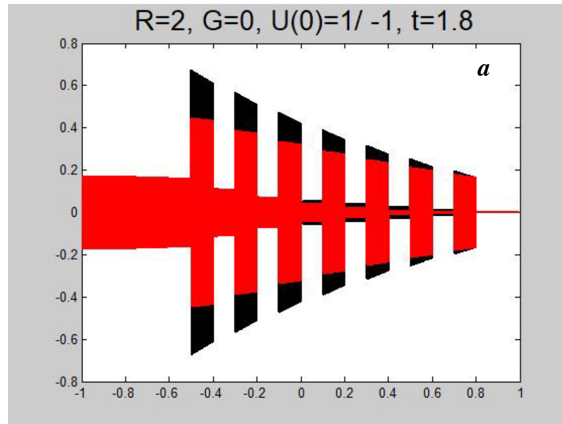
There is altogether another picture in the real lines that have losses, when only the direct electromagnetic wave is analyzed.

It can be seen that if the shape of the input impulse (beginning of the line) is bipolar according to (fig.2, c, d), then the spatial distribution of the electric current not so reflected differs from the function of the pulse voltage distribution function monopoly resulting from comparisons of these results.

If the polarity of the input signals will be changed to diametrically opaque, then for each time integration step: $u(0,t) = q(j) = f(j)\cdot(-1)(j+1)$;

where j = 1,2,3, ..., then the amount of electromagnetic energy brought in line will remain the same, but the structure of the electromagnetic wave field will essentially change both qualitatively and quantitatively according to (fig.3).

If the losses are essential in the real lines, so (R = 10; G = 0), then from the 7 (seven) very little steps that will remain.



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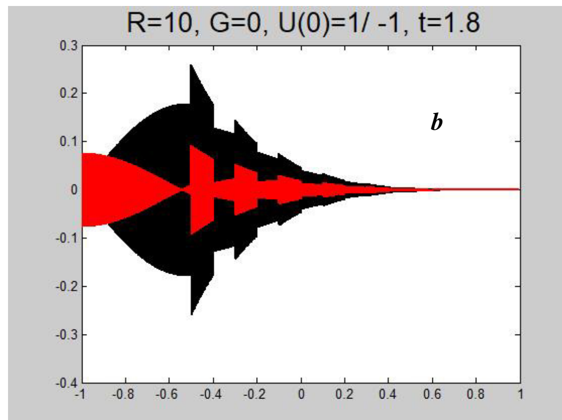


b

Fig. 3. The distribution of voltage (curve-1) and current (curve-2) in line at time t = 1.8 for a compact bipolar input impedance when R = 2; G = 0; 3, a for R = 10; G = 0 ,. fig.3,b.

The shape of the respective curves is essentially disparate, and the amplitude of the electric current becomes larger than the amplitude of the voltage, as seen in the previous analytical calculations. The analytical calculations of the tests presented with an accuracy of up to 3-4 digits can be obtained using the PaPuRi algorithm [10], which contains only four arithmetical operations.

This phenomenon can be confirmed if the program presented in the Matlab environment is presented below.

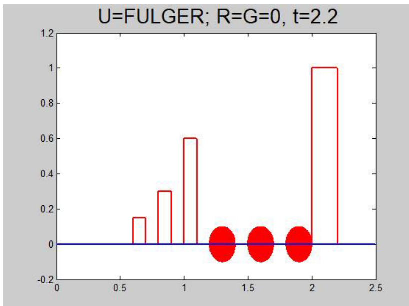
The wave propagation dynamics can be tracked in video footage on YouTube, which shows that the PaPuRi algorithm quite accurately describes not only the evolution of direct electromagnetic wave propagation (in direct time) but also the involution of electromagnetic waves reflected (indirectly) u scale error (masstable) of order 50 and higher, according to: (see., https://www.youtube.com/watch?v=g0o5ptcMD5k&t=42s).

No other algorithm and IT methods for solving telegraphic equations (1) can produce and repeat these results even for ideal lines $(G = R = 0)$.

According to [11], the most widely used FDTD analytical calculation method is devoted to over two thousand publications, does not meet these requirements with the above-determined conditions according to the above requirements: (see.,https://www.youtube.com/watch?v=g0o5ptcMD5k&t=42s).

If the spherical lightning strikes in the lightning conductor of different types of lines, then the way of time variation of the electric parameters are shown on (fig.4 and 5).

For the moment t = 2,2 (fig.4, a) if there is a hitting of the spherical lightning for the lossless line $(G = R = 0)$ it can be seen that the parameters of the light induced voltage vary but not essentially (fig.4, b).



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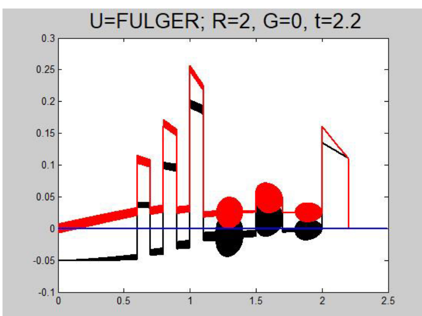

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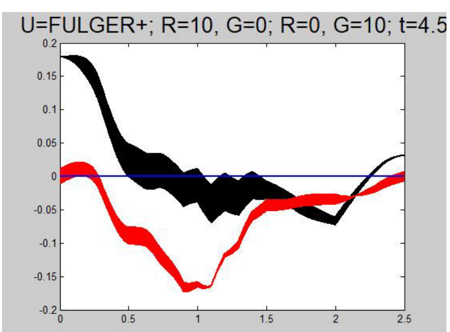
Fig.4.The way of lightning shape variation when struck in the lightning of the line does not have depressions (4, a, b);



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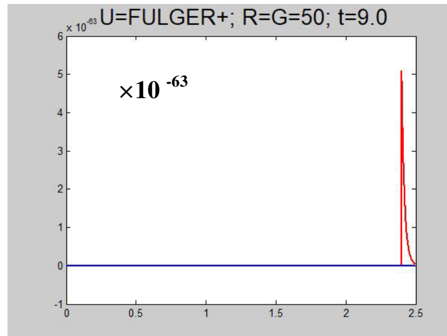

b

Fig.5. The mode of variation of the shape of the lightning strike in the lightning strike of the line having depressions (5, a, b);

But if the line possesses energy and voltage cuts, then, $(R = 10; G = 0)$, $(1 \prec x \prec 1,5)$ or $(R = 0; G = 10)$, $(2 \prec x \prec 2,5)$ then, for the time interval t = 4,5, the respective steps and balls have been made in somewhat time-varying values within fairly small limits according to fig. (5 a, b).

If $(R = G = 50)$, $[(1 \prec x \prec 1,5)/(2 \prec x \prec 2,5)]$ then for the time interval (t = 9) the signal form is transformed into a sharp triangle and moves in the parallel plane that exceeds the scale error (mashtar) with the order of 63.

Is it possible to carry that signal from that phase?

It is possible to use the PaPuRi algorithm described above.

The algorimt PaPuRi is anexed.

CONCLUSION

As a result of the analysis we can see that in the analytical calculation of the electromagnetic phenomena occurring in the lightning strikes of long electric transmission lines, the shape of the signals induced by lightning can have very different shapes.

The way of calculating both the direct electromagnetic waves and those of the electrified ones regardless of their lifetime can be done by using the PaPuRi algorithm.

REFERENCES

1. Krugh, K. A., Osnovî electrotehnici. –L.: ONTI, 1936, 888p.

2. Krugh, K. A., Perehodnîe gproțessî d lineinîh electriceschih țepeah.-M.-L.: GHEI, 1948, 344p.

3. Erhan, F. M., Bazele teoretice ale electrotehnicii. –Chișinău, UASM, 2009; 675p.

4. Bessonov, L.A. Teoreticeschie osnovî electrotehnichi. –M.: IURAIT,2013, 874p.

5. **Demircean, K.S.; Neiman, L R.; Korovkin, N.V.** Teoreticeschie osnovî electrotehnichi. T.2.-St Pb., 2009; Ediția 5; 432p.

6.Rîjov, Iu. P. Dalnie electroperedaci sverhvisokogho napreajenia. –M.: MEI, 2007. - 488p.

7. Butîrin, P. A. Teoreticeschie osnovî electrotehnichi. .-St Pb., 2009;Lani,2012.

8. Apollonschii, S. M. Differențialinîe uravnenia matematiceskoi phiziki v electrotehniche. –St. Pb., 2012.– 352 p.
9. Dragan, G., Golovanov, N., Mazzeti, C., Moraru, A., Nicoara, B., Nucci, C., Costea, M., Dragan L. Tehnica tensiunilor înalte. Vol. II. – Bucureşti: Editura AGIR, 2001. – 732p.
10. **Beril, S.I., Rimschii,V.K., Alhazov,A. E.** PaPuRi – Algoritm, teste, molnieotvod, shifrator.-Chişinău,USM,2018-264p.
11. **Vikipedia-**Metoda FDTD.

ANEXE

The algorimt PaPuRi

THE PRINSIPS FORMITEDE THE ALGORITM OF ANALYTICAL CALCULATION:

```
clear;
n=2500; nn=n+1; l=2.5; h=l/n; tau=h; k=4500;
R1=0; R2=0;
 for i=1:n
X(i)=i*h-h/2; R(i)=0; G(i)=0; D(i)=0; U(i)=0; UT(i)=0; end;
 for i=1001:1500
 R(i)=10; G(i)=0;  end;
 for i=2001:n
 R(i)=0; G(i)=10;  end;
 for i=1:n
al(i)=(R(i)+G(i))/2;
AD(i)=1+tau*al(i);
AD1(i)=1+tau*(al(i)-R(i));
AU1(i)=1+tau*(al(i)-G(i)); end;
```

CREATING INITIAL DATA FOR ANALYTICAL CALCULATION OF ELECTRIC CUTRENTS AND VOLTAGE:

```
 for i=1491:1710
 D(i+200)=(-1)^i*0.03; U(i+200)=-D(i+200);  end;
 for i=1576:1625
 D(i+200)=(-1)^i*0.1; U(i+200)=-D(i+200); end;
```

ANALYTICAL CALCULATION IN DIRECT REAL-ELECTROMAGNETIC DIRECTIONS:

```
 for j=1:k
   for i=2:n
   DA(i)=(D(i-1)+D(i)+U(i-1)-U(i))/2;
   UA(i)=(U(i-1)+U(i)+D(i-1)-D(i))/2; end;
```

DETERMINATION OF ENTRY VOLTAGE:

```
   T(j)=tau*j;
    UA(1)=1;
     if(j>200)
   T1(j)=T(j)-0.3;
   UA(1)=(-1)^j*abs((0.01-T1(j)^2))^0.5; end;
      if(j>400)
   UA(1)=0;  end;
     if(j>500)
    T1(j)=T(j)-0.6;
    UA(1)=(-1)^j*abs((0.01-T1(j)^2))^0.5;  end;
     if(j>700)
    UA(1)=0;   end;
    if(j>800)
    T1(j)=T(j)-0.9;
    UA(1)=(-1)^j*abs((0.01-T1(j)^2))^0.5; end;
     if(j>1000)
     UA(1)=0;   end;
     if(j>1100)
     UA(1)=0.6;   end;
     if(j>1200)
     UA(1)=0;  end;
    if(j>1300)
    UA(1)=0.3;   end;
    if(j>1400)
    UA(1)=0;  end;
    if(j>1500)
   UA(1)=0.15;  end;
   if(j>1600)
   UA(1)=0;  end;
   DA(1)=D(1)-U(1)+UA(1);
   DA(nn)=(D(n)+U(n))/(1+R2);
UA(nn)=(D(n)+U(n))*(R2/(1+R2));

     for i=1:n
   D(i)=(UA(i)-UA(i+1)+D(i)*AD1(i))/AD(i);
   U(i)=(DA(i)-DA(i+1)+U(i)*AU1(i))/AD(i); end;

 if(mod(j,10)==0)
 p=plot(X,D,'k',X,U,'r',X,UT,'b');
 set(p,'LineWidth',2);
title('\fontsize{20} U=FULGER; t=5.0');
pause(0.001);
 end; end;
 p=plot(X,D,'k',X,U,'r',X,UT,'b');
 set(p,'LineWidth',2);
title('\fontsize{20} U=FULGER+; R=10, G=0; R=0, G=10;
t=4.5');
pause(3);
fn=input('Save picture to filename [P]');
if(isempty(fn)) fn='P'; end;
ig=getframe(gcf); imwrite(ig.cdata,[fn '.jpg']);
```