THREE-PHASE TRANSMISSION CAPACITY LINES OF LENGTH λ/2, λ/4, λ/8, λ/16 AND THEIR FEATURES

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Abstract. The exact method of transient and steady-state calculating processes in multi-wire lines without losses under arbitrary initial and limited conditions is described. As examples, the results of calculations of the throughput of three-phase lines with wave lengths (λ/2, λ/4, λ/8, and λ/16) in the running mixed waves with separated and converged phase wires are given.

Keywords: three-phase alternating current power transmission lines, telegraph equations, method of characteristics, transient and steady-state processes in alternating current electrical circuits, generated and transmitted power, mutual inductance of phase wires (M), inductance matrix(L), capacitance matrix (C) and wave resistance (Z_U).

INTRODUCTION

Three-phase systems of electrical energy transmission and power supply to consumers is the greatest invention of mankind, but, relatively are poorly studied. In textbooks on power supply, theoretical foundations of electrical engineering, and other sources and special literature, there is no basic information about the exact quantitative characteristics of power transmission processes on three-phase alternative current power lines of various wave lengths (λ).

For a sinusoidal voltage source with an industrial frequency *f*=50 Hz, the wavelength λ is approximately λ =6000 km, which the electromagnetic wave runs in Δτ=20ms. For the query "Capacity of alternating current power transmission lines" in the technical literature, this interpretation is very common.

The capacity of alternating current power transmission lines is determined by the amount of active power that the line can transmit when all the conditions that determine its normal operation are met.

The natural capacity of cable lines is an order of magnitude greater than that of overhead lines. The transmitted power depends on the line length (l), wave characteristics (wave resistance (Z_U), and phase change coefficient). In the special literature [1-3, 7], we can find a lot of general arguments that power transmission lines with a length close to half-wave have a number of undeniable advantages over all others.

No less interesting is the length (λ / 8), which according to the" anatomy " of the alternating current line can not transmit more than one power rating [1-3].

Much is also said about the effect of increasing the natural current due to the convergence of phase wires. However, there are no specific numerical data on this subject, although according our opinion they should appear in the educational and reference literature.

The relationship between the maximum possible power (natural) transmitted over such lines, (throughput) and the maximum length of overhead power lines starting from the voltage U-35 kV and above are shown in tab. 1.

Table 1

The voltage, of kV .	Cross section wire, mm^2	The transmitted power, MW		The power line length, km	
		The maximum limit natural power	The maximum power at a current density of j=1,1 A/mm^2	The maximum power with an efficiency factor of 0.9	The average distance between two adjacent transformer substations
35	70…150	3	4…10	25	8
110	70…240	30	13…45	80	25
150	150…300	60	38…77	250	20
220	24…400	135	90…150	400	100
330	**2·240…2·400**	**360**	**270…450**	**700**	**130**
500	3·300…3·500	900	770…1300	1200	280
750	5·300…5·500	2100	1500…2000	2200	300
1150	**8·300…8·500**	**5200**	**4000…6000**	**3000**	**—**

Capacity and power transmission range in power lines with voltages of 35 ... 1150 kV

Therefore, it is obvious interest to determine the dependence of the transmitted power not only on the length of the three-phase power transmission, but also its wave resistance (Z_U), which is no longer a scalar value as in the case of a single-wire model, but is a matrix represented by equation (1).

$$Z_U = \sqrt{L/C} \tag{1}$$

where L and C are symmetrical matrices of proper and mutual inductors (L) and capacities (C) of a three-phase power transmission line, regardless of the voltage level.

Since the three-phase system is not presented as a single-line circuit, it must contain both its own and mutual inductors (L) and the corresponding capacities (C), so these parameters of three-phase systems in real mode are represented as matrices.

1. STATEMENT OF THE PROBLEM.

To solve the problem outlined above, there is no other way than to integrate Telegraph equations under appropriate initial and limited conditions.

To expose the essence of the issue as much as possible, we formulate the initial boundary value problem for a long power transmission line without losses $|R| = |G| = 0$.

The beginning of the line corresponds to the (x=0) coordinate of the sinusoidal voltage source, which is modeled by the limited conditions represented by equation (2).

$$\begin{aligned} u_1(0,t) &= U_0 \sin(\omega \cdot t) = U_0 \sin(2\pi f \cdot t) \\ u_2(0,t) &= U_0 \sin(\omega \cdot t - 2\pi/3) = U_0 \sin(2\pi f \cdot t - 2\pi/3) \\ u_3(0,t) &= U_0 \sin(\omega \cdot t - 4\pi/3) = U_0 \sin(2\pi f \cdot t - 4\pi/3) \end{aligned} \tag{2}$$

A single matrix (3) will be used to analyze physical phenomena in three-phase power lines.

$$v^2[C]\cdot[L] = [E] \tag{3}$$

where: v - is the speed of electromagnetic waves distribution of voltage and current. In this article, we assume that the of electromagnetic distribution waves of voltage and current is identical in all three phases, since the load is symmetrical.

At the end (at the receiving end) of the line *(x =l),* we set the expected value for the active load (R) as instantaneous expected values.

For this mode, the instantaneous value of the voltage at any time to any point of the line *(l)* is determined analytically according to equation (4).

$$\vec{u}(l,t) = R_l \vec{i}(l,t) \tag{4}$$

If equation (4) is presented as a coordinate entry for a three-phase system, it will take the form of expression (5), for any point along a three-phase power line, both for the beginning of the line ($x = l = 0$) and for the end of the line ($x = l$) at the consumer, which can determine the value of the line voltage at any time (t).

$$\begin{aligned} \vec{u}_1(l,t) &= R_{11}\vec{i}_1(l,t) + R_{12}\vec{i}_2(l,t) + R_{13}\vec{i}_3(l,t) \\ \vec{u}_2(l,t) &= R_{21}\vec{i}_1(l,t) + R_{22}\vec{i}_2(l,t) + R_{23}\vec{i}_3(l,t) \\ \vec{u}_3(l,t) &= R_{31}\vec{i}_1(l,t) + R_{32}\vec{i}_2(l,t) + R3_{13}\vec{i}_3(l,t) \end{aligned} \tag{5}$$

For a symmetrical load of a three-phase system, regardless of the connecting method the load "star" or "triangle", when conditions (5) are met, the load resistance value (R_S) changes depending on the considered and simulated short-circuit or idling mode. When modeling the short-circuit mode of a three-phase line, the load resistance fulfills the condition ($R_S = 0$), and using the idle mode, the load resistance fulfils ($R_S = \infty$), while proceeding from the fulfillment of the symmetry phase resistances, conditions and the mutual phase loads are ignored. In this case, the conditions (6) are met

$$\begin{aligned} R_{11} &= R_{22} = R_{33} = R_S \\ R_{12} &= R_{21} = R_{13} = R_{31} = R_{23} = R_{32} = 0 \end{aligned} \tag{6}$$

In normal operation of a three-phase system, the maximum power is transferred from the source to the load at a matched load when condition (7) is met.

$$Z_V = R_S \tag{7}$$

When condition (7) is met, the transmission line implements a running wave mode: full absorption of all energy supplied to the load (all energy is transmitted to the load by a direct wave), this mode has no reflected electromagnetic waves and conditions (8) are met.

$$Z_V = R_S = R_l = \sqrt{L/C} = L^{1/2} \cdot C^{-1/2} \tag{8}$$

Since the load is connected simultaneously to three phases, then according to Kirchhoff's laws, one should set such limited relations that correspond to the conditions (9).

$$u_1(l,t) = u_2(l,t) = u_3(l,t) = u(l,t),$$
$$i(l,t) = i_1(l,t) + i_2(l,t) + i_3(l,t),\ u(l,t) = R_S \cdot i(l,t). \tag{9}$$

2. THE METHOD OF POINT FEATURES.

The method of point characteristics proceeds from an analytical representation of an ideal power transmission line using for this purpose telegraphic equations presented according to (10).

$$L\frac{\overline{\partial i}}{dt}+\frac{\overline{\partial u}}{dt}=0;\frac{\overline{\partial i}}{dt}+C\frac{\overline{\partial u}}{dt}=0 \qquad (10)$$

If we multiply scalar the first equation of the system (10) by (v^2C), and the second – by (v), we get a system of equations (11).

$$v^2CL\frac{\partial\vec{i}}{\partial t}+v^2C\frac{\partial\vec{u}}{\partial x}=0;vC\frac{\partial\vec{u}}{\partial t}+v\frac{\partial\vec{i}}{\partial x}=0. \qquad (11)$$

If we enter notation $\vec{I}^{\pm}\equiv\vec{i}\pm vC\vec{u}$, these differential expressions (11) can be represented as equation (12).

$$\frac{\partial\vec{I}^{\pm}}{\partial t}\pm v\frac{\partial\vec{I}^{\pm}}{\partial x}=0\,. \qquad (12)$$

From the general solution of the form ($\vec{I}^{\pm}=\vec{\varphi}(x\mp v\cdot t)$), it follows that the introduced.

Riemann invariants preserve constant values ($\vec{I}^{\pm}\equiv\vec{i}\pm vC\vec{u}=const.$) along lines ($dx/dt=\pm v$), called characteristics of a hyperbolic system of differential equations (12).

To calculate the desired functions at the edge point (x = 0 and x = l), we use invariants ($\vec{I}^{-}\equiv\vec{i}-vC\vec{u}$) that preserve constant values along a family of lines with a negative slope ($dx/dt=-v$).

Together with three given limited conditions, they form a system of six algebraic equations with six unknowns at any point of time discretization on a straight line (x = 0).

In the same way, the invariant ($\vec{I}^{+}\equiv\vec{i}+vC\vec{u}$) is used to find the time values of currents and voltages at the receiver (consumer).

It follows that the mode of running waves, when the resistance of the receiver (R_S). is consistent with the resistance of the line (Z_U) and there are no reflected waves (are not generated) in the system, can be implemented only if condition (8) is met, which is graphically shown in fig.(1).

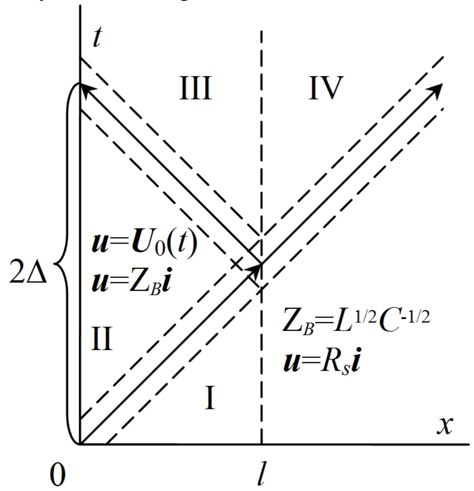


Fig. 1. Configuration of wave fronts on the (*xt*) plane in a line of length *l*.

The characteristics method, being the main component of the PaPuRi algorithm, is mainly illustrative.

In particular, it allows you to get a formula for determining the input currents in a multi-wire line when the input voltages are known: $\vec{i}=vC\vec{u}$.

3. THE RESULTS OF COMPUTATIONAL EXPERIMENTS WITH TRANSMISSION OF THREE-PHASE LINES OF DIFFERENT LENGTHS.

To justify the value of the transmitted power (P_0) for three-phase altering current, regardless of the form of execution, but depending on the length of the line (l), analytical calculations were performed for lines with different wave lengths, namely (λ/2, λ/4, λ/8, λ/16), the results are presented analytically and graphically in various fig. (2-10).

Since any steady-state regime is always preceded by a non-stationary wave process, their calculation should be carried out within the framework of a single approach using uniform formulas in the same sequence as in reality.

The steady-state or quasi-steady distribution of current and voltage in an electrical circuit must be obtained as a consequence of a stationary transient process and in no other way.

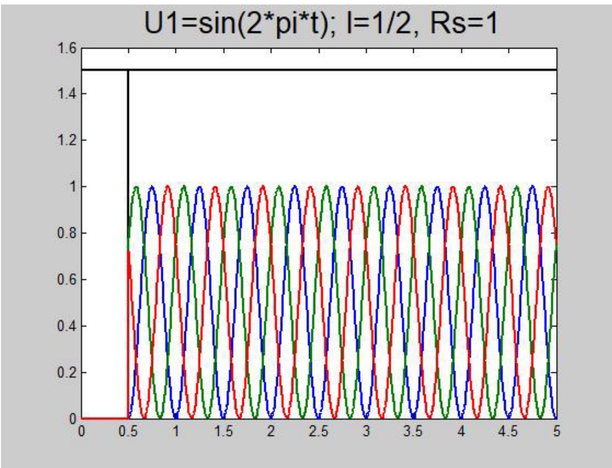
Let's first consider three-phase lines without taking into account the mutual influence of the physical parameters of the phase wires, for which condition (13) is fulfilled in order to establish the dependence of the transmitted power on the value of the load resistance.

$$|L|=|C|=|E| \qquad (13)$$

For graphical interpretation of the dynamic\`s of changes in the generated and transmitted power for a half-wave line ($l=\lambda/2$), at different resistance values (for the following resistance values R_S = 1; 1/5; 5), which allows you to immediately detect the inversely proportional dependence of the transmitted power on the variable parameter of the transmitted power within P1= 1.5: 7.5; 0.3.of the fig. (2-10) a special program was developed in Matlab.

For graphical interpretation of the dynamics of changes in the generated and transmitted power for a half-wave line (l=λ/2), at different resistance values (for the following resistance values R_S = 1; 1/5; 5), which allows you to immediately detect the inversely proportional dependence of the transmitted power on the variable parameter of the transmitted power within P1= 1.5: 7.5; 0.3.

On a fig. (2,a,b,c) the dynamics of change of the generated and transferrable power is presented for the semi wave line of electricity transmission ($l=\lambda/2$), at the next values of resistance of loading of R_S = 1; 1/5; 5, that allows at once to find out inversely proportional dependence of transferrable power on the varied parameter: P_1= 1.5 : 7.5; 0.3.



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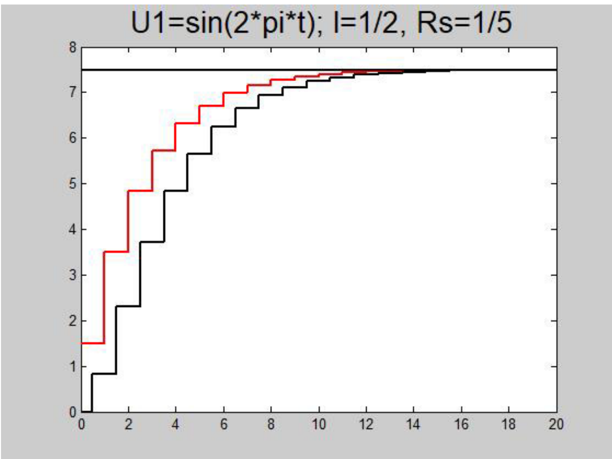

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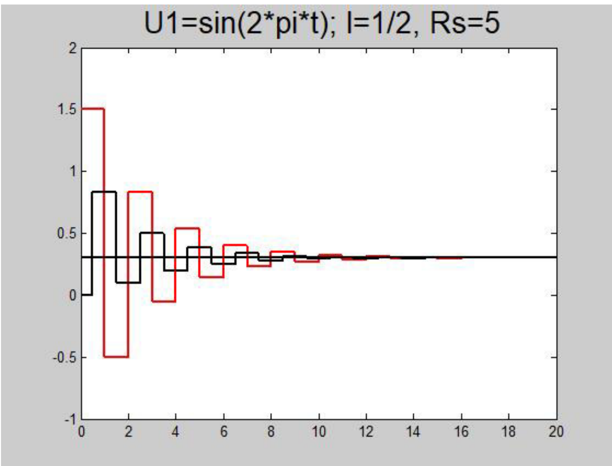


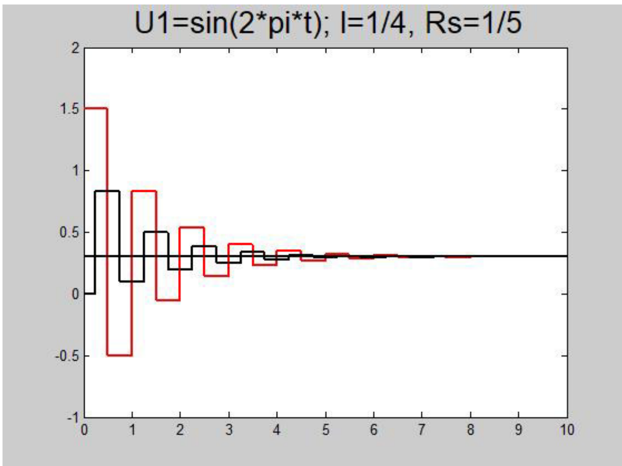
c

Fig. 2. Dynamics of changes in generated and transmitted power for a half-wave line l = λ /2 for the following load resistance values: R_S = 1; 1/5; 5.

If you take the nominal power P_0=1.5, then for an ideal transmission length (l = λ /2, λ 1, 3 λ /2, 2λ), so we get the formula P_1 (R_S) = P_0 /R_S

Thus, unlimited power can be transmitted over lines with a wave length multiple of (λ/2) in a conditions close to the short-circuit mode: $R_S \to 0$.

For a quarter-wave transmission line *(l=λ/4)*, a directly proportional dependence of the transmitted and consumed power on the load resistance is obtained: $P_1(R_S) = P_0 x R_S$, which is shown in fig. (3,a,b).



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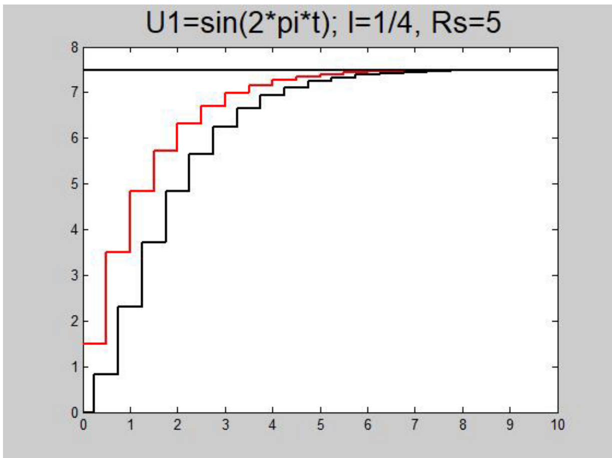

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Fig. 3. Dynamics of changes in the generated and transmitted power for a quarter-wave line *(l=λ /4)*, with load resistance values: R_S =1/5; 5

For a load value of R_S =1/5; 5, the exact same power values are obtained as for R_S = 5; 1/5 for a half-wave line *(l=λ/2)*.

Thus, and on an ideal quarter-wave line *(l=λ/4)*, it is possible to transmit arbitrarily large power, but already in a conditions close to idling: $R_S \to \infty$.

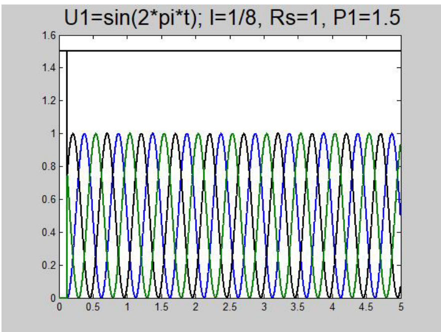
Let us now turn to a line with a wave length equal to *(l=λ / 8)*.

For this variant, shown in fig. (4), we are waiting for the first not quite pleasant surprise in the form of an equation of the type: P_1 (R_S) = $P_1(1/R_S)$.

What would it be?

In principle, it is impossible to transmit more than one power rating over a transmission line with a length equal to *(l=λ/8)*, that is, half the length of a quarter-wave line *(l=λ/4)*, which is achieved only in the conditions of traveling waves $P_1 = P_0 = 1.0$.

The same "dead" load nodes are equal lengths (l = λ /8; 3 λ /8; 5 λ /8,) as indicated in [1-3,6].



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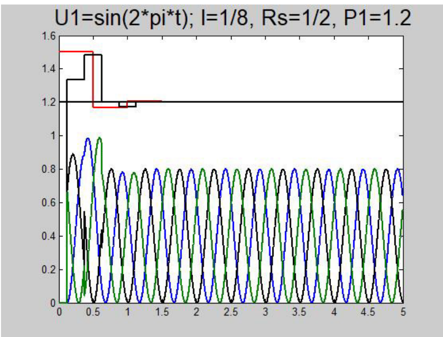

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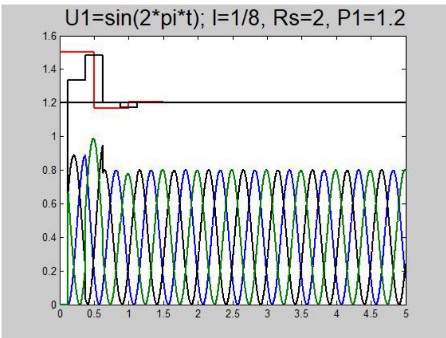

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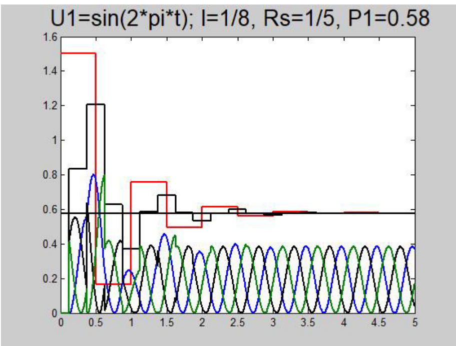

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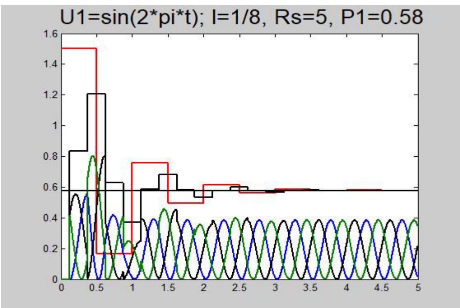


e

Fig. 4. Dynamics of changes in generated and transmitted power for a line of length *(l = λ /8)* for load resistance values: R_S =1; 1/2; 2; 1/5; 5.

What would it be?

In principle, it is impossible to transmit more than one power rating over a transmission line with a length equal to *(l=λ/8),* that is, half the length of a quarter-wave line *(l=λ/4),* which is achieved only in the conditions of traveling waves $P_1 = P_0 = 1.0$.

The same "dead" load nodes are equal lengths (l = λ /8; 3 λ /8; 5 λ /8,) as indicated in [1-3,6].

When performing repeated calculations, but taking into account the mutual influence of horizontally located wires and their mutual inductance (M) , set the matrix of the three-phase line's reactive parameters in the form of ratios (11 and 12)

$$C = \begin{pmatrix} 31.98 & -22.99 & -4.04 \\ -22.99 & 50.90 & -22.99 \\ -4.04 & -22.99 & 31.98 \end{pmatrix} \text{ nF/km;}$$

$$Z_B = \begin{pmatrix} 281 & 212 & 188 \\ 212 & 257 & 212 \\ 188 & 212 & 281 \end{pmatrix} \Omega; \tag{11}$$

$$C = \begin{pmatrix} 2.64 & -1.90 & -0.33 \\ -1.90 & 4.21 & -1.90 \\ -0.33 & -1.90 & 2.64 \end{pmatrix} \text{ nF/km}$$

$$Z_B = \begin{pmatrix} 1.01 & 0.76 & 0.68 \\ 0.76 & 0.93 & 0.76 \\ 0.68 & 0.76 & 1.01 \end{pmatrix} .\Omega \tag{12}$$

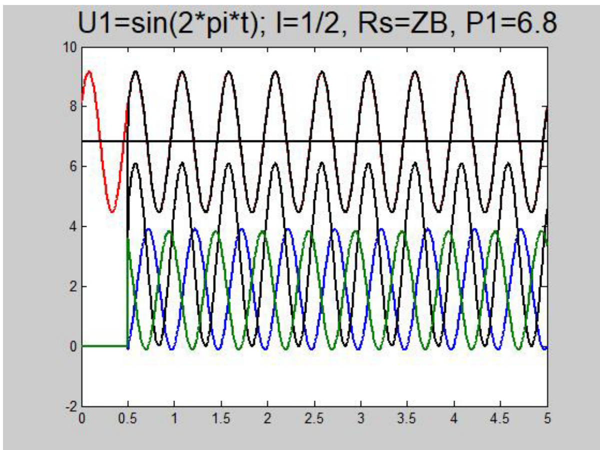
When performing repeated calculations, but taking into account the mutual influence For a single-wire model of a power line, the technical parameters take the values:

C= 12.10 nF/km, $L = 0.9348$ mG/km,

$a = 297\ 336$ km/s, $Z_B = 278\ \Omega$; $C = L = a = Z_B = 1$.

The numerical values of the matrices were calculated fairly accurately based on the solution of the field problem for the equations of electrostatics [4].

For fig. (5-8) shows graphs of the time dependence of generated and transmitted power for three-phase lines of length *(l =λ/2; λ/4; λ/8; λ/16)* with converged phases of length for two variants of load resistances: $R_S = Z_B = 1$.



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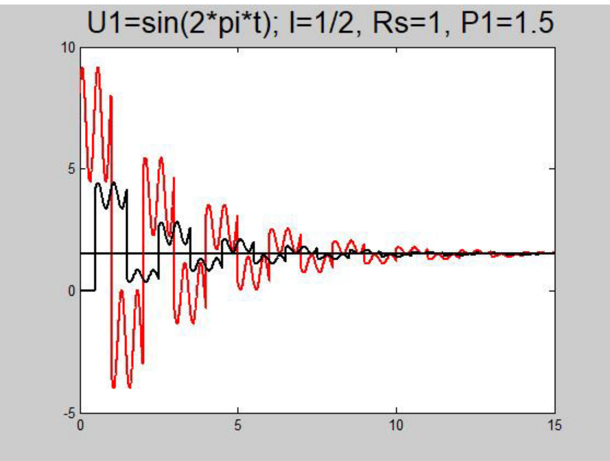

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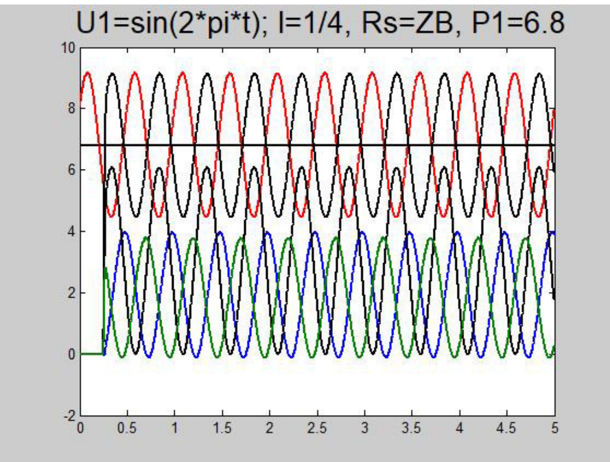
Fig. 5. Dynamics of changes in generated and transmitted power for a half-wave line (l = λ/2) with converged phases at load resistance values: $R_S = Z_B$; 1



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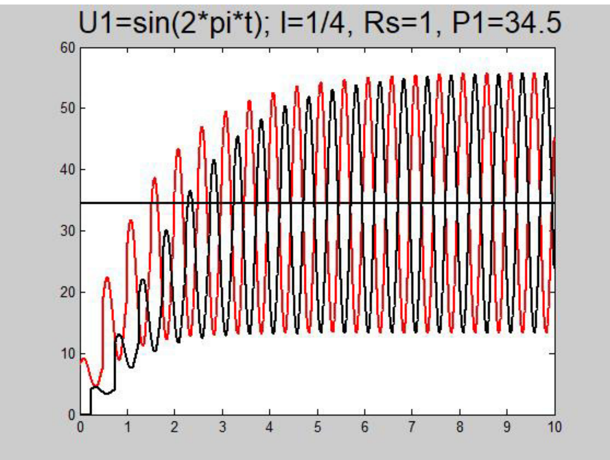

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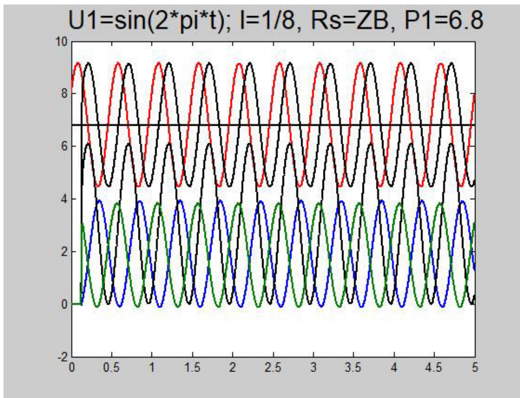
Fig. 6. Dynamics of changes in the generated and transmitted power for a quarter-wave line *(l = λ/4)* with close phases at the values of the load resistance $R_S = Z_B$; 1.



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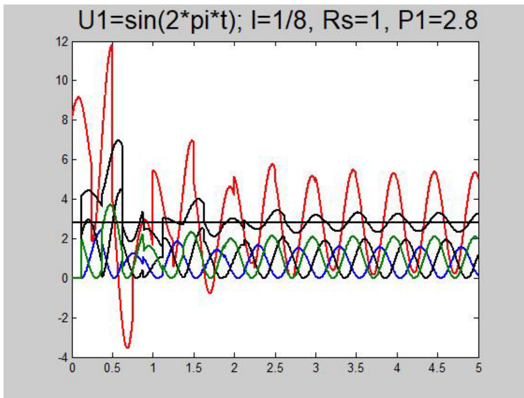

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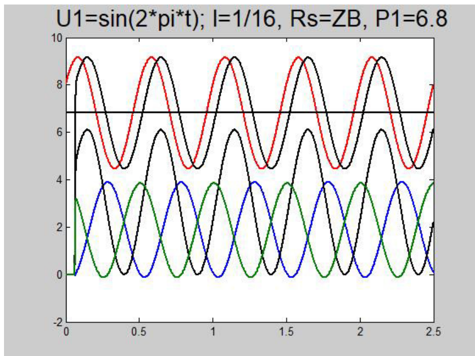
Fig. 7. Dynamics of changes in generated and transmitted power for a line of length (l = λ/8) with converged phases at load resistance values: $R_S = Z_B$; 1.



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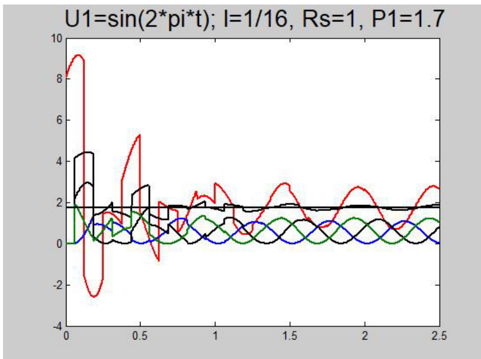

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Fig. 8. Dynamics of changes in generated and transmitted power for a line of length (l = λ/16) with close phases at load resistance values: $R_S = Z_B$; 1

When the power line is closed to the agreed load $R_S = Z_B$, identical results are obtained in all four cases: $P_1(Z_B)$ =4.55; P_0=6.83.

The received values of transmitted power show that-the maximum possible transmitted power is more than four and a half ratings.

If for determining the above initial parameters of three-phase power lines are approximate formulas, not taking into account the geometry of the cross section of the wires [5-7], the numerical values of reactive parameters of three-phase transmission line presents the calculated matrices in the form of ratios take the numerical values are represented by the matrices (13, 14)

$$C = \begin{pmatrix} 31.98 & -22.99 & -4.04 \\ -22.99 & 50.90 & -22.99 \\ -4.04 & -22.99 & 31.98 \end{pmatrix} \text{ nF/km;}$$

$$Z_B = \begin{pmatrix} 281 & 212 & 188 \\ 212 & 257 & 212 \\ 188 & 212 & 281 \end{pmatrix} \Omega; \qquad (13)$$

$$C = \begin{pmatrix} 2.64 & -1.90 & -0.33 \\ -1.90 & 4.21 & -1.90 \\ -0.33 & -1.90 & 2.64 \end{pmatrix}; \text{ nF/km}$$

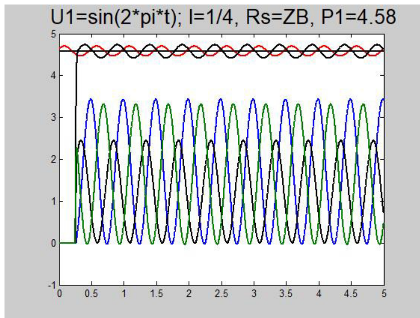
$$Z_B = \begin{pmatrix} 1.01 & 0.76 & 0.68 \\ 0.76 & 0.93 & 0.76 \\ 0.68 & 0.76 & 1.01 \end{pmatrix} \Omega \qquad (14)$$

In this case, analytical calculations show that the maximum possible transmitted power differs from the ideal version, but also amounts to more than four and a half ratings and, accordingly, is equal to $P_1(Z_B)$ = 4.03; P_0 = 6.04.

As we can see, the error is not so significant (fatal) and it can be ignored when performing engineering calculations, since it does not exceed 5%, as we can see, the error is not so significant (fatal) and it can be ignored when performing engineering calculations, since it does not exceed 5%, (Δ<5%.).

The power fluctuations can be significantly reduced if, for example, for a quarter-wave line *($l = \lambda/4$)*, the value of the phase voltage of the middle wire is reduced by 1.75 times for $R_S = Z_B$ and 4 times for R_S =1, but, of course, the average power value for the period is also reduced .

These calculations are graphically presented in fig. (9).



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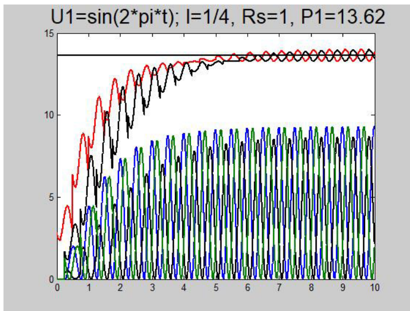

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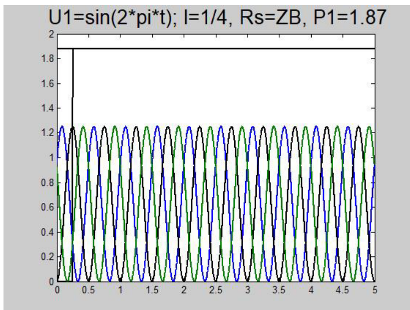
Fig. 9. Dynamics of changes in generated and transmitted power for a line of length *($l = \lambda / 4$)* with converged phases when the phase voltage of the middle wire is reduced by 1.75 and 4 times

If the phases of a power transmission line are arranged in a triangle, the matrix of proper and mutual capacitances of such a line and wave resistances can be represented by the equations (15).

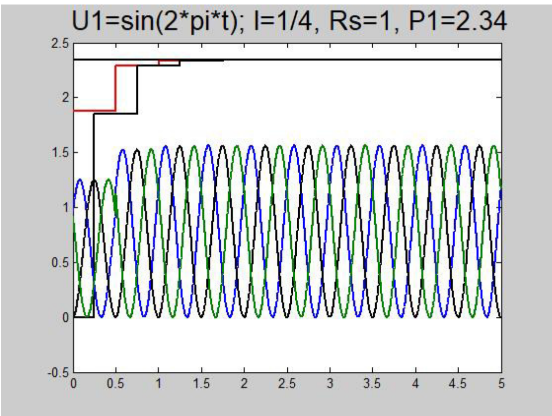
$$C = \begin{pmatrix} 1 & -0.25 & -0.25 \\ -0.25 & 1 & -0.25 \\ -0.25 & -0.25 & 1 \end{pmatrix} \text{ nF/km} \quad ;$$

$$Z_B = \begin{pmatrix} 1.2 & 0.4 & 0.4 \\ 0.4 & 1.2 & 0.4 \\ 0.4 & 0.4 & 1.2 \end{pmatrix} \Omega. \qquad (15)$$

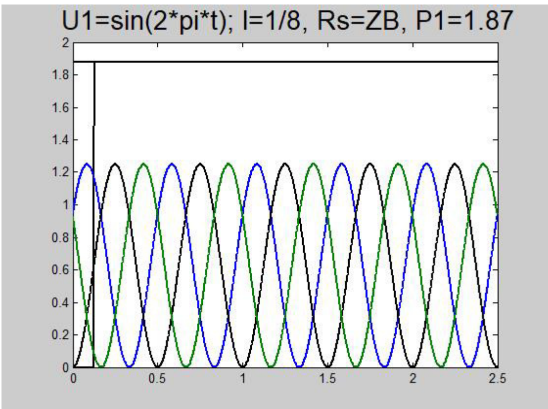
In this case, the effect of increasing the transmitted power due to the converged phases is significantly weakened, as can be seen from the analytical results presented in fig. (10) weakened, as can be seen from the analytical results presented in fig. (10).



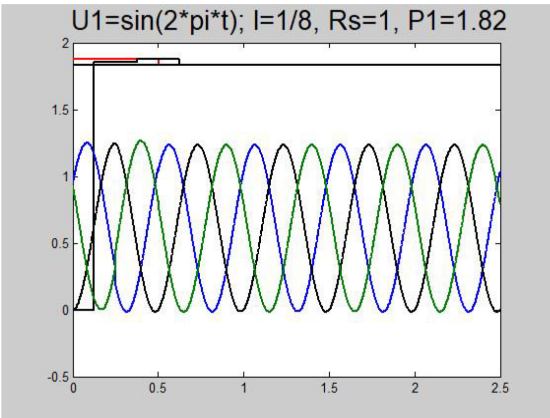
a



b



c



d

Fig. 10. Dynamics of changes in generated and transmitted power for lines of length ($l = \lambda / 4$) and ($l = \lambda / 8$) with a triangular arrangement of phase wires.

4. CONCLUSIONS

A preliminary parametric analysis of the influence of the length of a three-phase power transmission line and its load resistance on the transmitted power in transient and steady-state modes is carried out.

The question immediately arises how to implement the traveling wave conditions in practice, so that all the electromagnetic energy supplied to the load is completely absorbed by it. This requires at least two consumers, one of which must be connected according to the "star" scheme, and the second according to the "triangle" scheme.

To solve the problems considered, taking into account the losses in the line, it is already necessary to use the PaPuRi algorithm [4], which gives as accurate results as the method of point characteristics.

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